Ispitivanje f-je

Ispitati f-ju znači odrediti

a) oblast definisanosti

b) parnost (neparnost) i periodičnost

c) nule, presjek grafa sa Y-osom, znak f-je

d) ponašanje na krajevima intervala definisanosti i asimptote

e) rast i opadanje f-je (intervale u kojima f-ja raste ili opada)

f) ekstreme f-je (minimum i maksimum ako ih ima)

g) prevojne tačke i intervale konveksnosti i konkavnosti

h) na osnovu svega ovoga nacrtati graf

Definiciono područje obilježavat ćemo sa $\mathcal{D}$ i to je skup svih onih vrijednosti u kojima je f-ja definisana (ima konačnu ili beskonačnu vrijednost).

1. Odrediti definiciono područje sljedećih f-ja:

a) $y=\frac{1}{x}$, Rj. $\mathcal{D}: \mathbb{R}\setminus\{0\}$ ili $\mathcal{D}: x\in(-\infty,0)\cup(0,+\infty)$

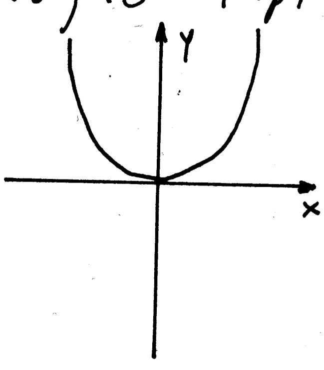
b) $y=\sqrt{x}$, Rj. $\mathcal{D}: x\in\mathbb{R}_0^+$ ili $\mathcal{D}: x\in[0,+\infty)$ ili $\mathcal{D}: x\geq 0$

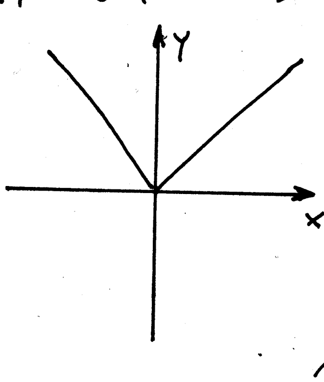
c) $y=\log x$, Rj. $\mathcal{D}: x\in\mathbb{R}^+$ ili $\mathcal{D}: x\in(0,+\infty)$ ili $\mathcal{D}: x>0$

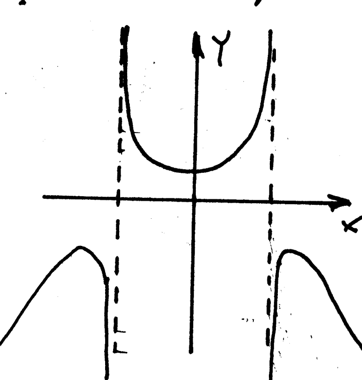
d) $y=\frac{1}{\sqrt{x}}$, Rj. $\mathcal{D}: x\in\mathbb{R}^+$ ili $\mathcal{D}: x\in(0,+\infty)$ ili $\mathcal{D}: x>0$

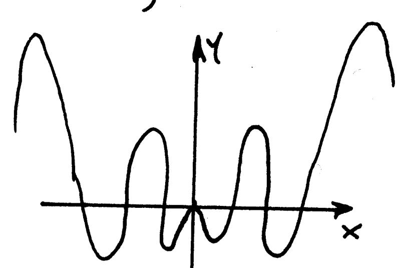
e) $y=\frac{\log x}{x-2}$, Rj. $x>0$, $x-2\neq 0$, $\mathcal{D}: x\in\mathbb{R}^+\setminus\{2\}$ ili $\mathcal{D}: x\in(0,2)\cup(2,+\infty)$

F-ja je parna ako je $\forall(x\in\mathcal{D})\ f(-x)=f(x)$. Grafik parne f-je je simetričan u odnosu na Y-osu i f-ju je dovoljno ispitati za $x\geq 0$. Grafici parnih f-ja:

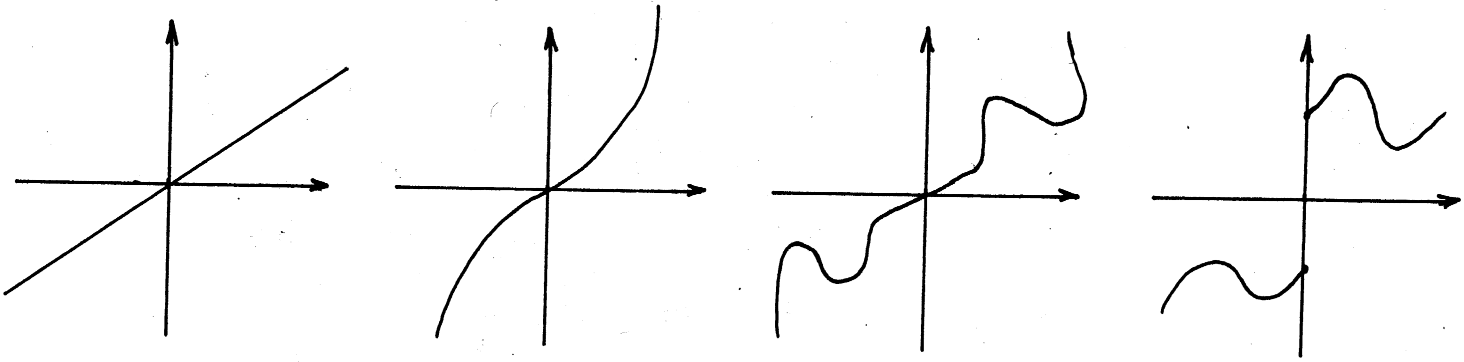








Ako je $\forall (x \in D)$ $f(-x) = -f(x)$ f-ja $f(x)$ je neparna f-ja.
Grafik neparne f-je je simetričan u odnosu na koordinatni početak $(0,0)$ pa je f-ju dovoljno ispitati za $x \geq 0$.
Grafici neparnih f-ja:



(2.) Odrediti parnost i neparnost sljedećih f-ja

a) $y = \frac{x^3}{x^2-4}$ Rj. $f(-x) = \frac{(-x)^3}{(-x)^2-4} = \frac{-x^3}{x^2-4} = -\frac{x^3}{x^2-4} = -f(x)$

f-ja je neparna

b) $y = \frac{x^2+1}{\sqrt{x^2-1}}$ Rj. $f(-x) = \frac{(-x)^2+1}{\sqrt{(-x)^2-1}} = \frac{x^2+1}{\sqrt{x^2-1}} = f(x)$ f-ja $f(x)$ je parna

c) $y = \frac{(x+1)^3}{(x-1)^2}$ Rj. Parnost i neparnost ima smisla ispitati samo ako je D simetrično. U našem slučaju $D: (-\infty, 1) \cup (1, +\infty)$ nije simetrično pa f-ja nije ni parna ni neparna.

II način: $f(-x) = \frac{(-x+1)^3}{(-x-1)^2}$ $\Rightarrow$ f-ja nije ni parna ni neparna

---

Neka je data f-ja $y = f(x)$.
Ako je za svako $x \in (a,b)$ $y'(x) < 0$ tada f-ja y opada ($\searrow$) na (a,b)
Ako je za svako $x \in (a,b)$ $y'(x) > 0$ tada f-ja y $\nearrow$ (raste) na (a,b)
Rješenjem jednačine $y' = 0$ dobijamo stacionarne tačke $x_1, x_2, \ldots, x_n$ koje konkurišu za ekstrem. Stacionarne tačke $x_1, x_2, \ldots, x_n$ mogu ali i ne moraju da budu tačke u kojima f-ja poprima ekstrem. Da li je stacionarna tačka x_1 ekstrem možemo zaključiti na dva načina:

I način: Na osnovu tabele rasta i opadanja,
$\overset{x_1}{\nearrow \searrow}$ MAX $\quad \overset{x_1}{\searrow \nearrow}$ MIN

II način: x_1 je stacionarna tačka

Ako je $y''(x_1) < 0$ $\Rightarrow$ $(x_1, f(x_1))$ je tačka u kojoj f-ja y ima maksimalnu vrijednost

Ako je $y''(x_1) > 0$ $\Rightarrow$ $(x_1, f(x_1))$ je tačka u kojoj f-ja y ima minimalnu vrijednost

3. Nađi ekstreme i intervale rasta i opadanja sljedećih f-ja:

a) $y = \dfrac{x^3}{x^2-4}$ Rj. $\mathcal{D}: x \in (-\infty, -2) \cup (-2, 2) \cup (2, +\infty)$

$$y' = \left(\frac{x^3}{x^2-4}\right)' = \frac{3x^2(x^2-4) - x^3 \cdot 2x}{(x^2-4)^2} = \frac{x^2(3x^2-12-2x^2)}{(x^2-4)^2} = \frac{x^2(x^2-12)}{(x^2-4)^2}$$

$y''=0$ ako i samo ako $x^2=0$ ili $x^2-12=0$

$x=0$ ili $x_{1,2} = \pm\sqrt{12}$ tj. $x_{1,2} = \pm 2\sqrt{3}$

Stacionarne tačke su $x_1 = 0$, $x_2 = -2\sqrt{3}$ i $x_3 = 2\sqrt{3}$.

prekidi f-je y + nule f-je y'

$-2\sqrt{2}$ -2 0 2 $2\sqrt{2}$

rast i opadanje

x	$(-\infty, -2\sqrt{3})$	$(-2\sqrt{3}, -2)$	$(-2, 0)$	$(0, 2)$	$(2, 2\sqrt{3})$	$(2\sqrt{3}, +\infty)$
y'	+	−	−	−	−	+
y	↗	↘	↘	↘	↘	↗

MAX MIN

$$f(-2\sqrt{3}) = \frac{(-2\sqrt{3})^3}{(-2\sqrt{3})^2-4} = \frac{-24\sqrt{3}}{8} = -3\sqrt{3}$$

$$f(2\sqrt{3}) = \frac{24\sqrt{3}}{12-4} = 3\sqrt{3}$$

$M(-2\sqrt{3}, -3\sqrt{3})$ je tačka lokalnog maksimuma a tačka $N(2\sqrt{3}, 3\sqrt{3})$ je tačka lokalnog minimuma

b) $y = \dfrac{x^2+1}{\sqrt{x^2-1}}$ Rj. $\mathcal{D}: x \in (-\infty, -1) \cup (1, +\infty)$

$$y' = \frac{2x\sqrt{x^2-1} - (x^2+1)\cdot\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2-1}}}{x^2-1} = \frac{2x(x^2-1) - x(x^2+1)}{(x^2-1)\sqrt{x^2-1}} = \frac{x(2x^2-2-x^2-1)}{(x^2-1)\sqrt{x^2-1}}$$

$y' = \dfrac{x(x^2-3)}{(x^2-1)\sqrt{x^2-1}}$, $y'=0$ akko $x=0$ ili $x^2-3=0$

$x_{1,2} = \pm\sqrt{3}$

Stacionarne tačke su $x_1=0$, $x_2 = -\sqrt{3}$ ili $x_3 = \sqrt{3}$.

prekidi f-je y + nule f-je y'

$-\sqrt{3}$ -1 0 1 $\sqrt{3}$

x	$(-\infty, -\sqrt{2})$	$(-\sqrt{3}, -1)$	$(-1, 0)$	$(0, 1)$	$(1, \sqrt{3})$	$(\sqrt{3}, +\infty)$
y'	−	+			−	+
y	↘	↗			↘	↗

rast i opadanje

MIN MIN

Tačke $M(-\sqrt{3}, 2\sqrt{2})$ i $N(\sqrt{3}, 2\sqrt{2})$ su tačke lokalnog minimuma.

$$f(-\sqrt{3}) = \frac{3+1}{\sqrt{3-1}} = \frac{4}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}, \quad f(\sqrt{3}) = 2\sqrt{2}$$

4. Ispitati i grafički predstaviti f-ju $Y = \dfrac{x}{x-3}$.

Rj. definiciono područje

$x - 3 \neq 0$

$x \neq 3$

$\mathcal{D}: (-\infty, 3) \cup (3, +\infty)$

parnost (neparnost), periodičnost

$\mathcal{D}$ nije simetrično $\Rightarrow$

$\Rightarrow$ f-ja nije ni parna ni neparna.

F-ja $f(x)$ je periodična sa periodom T ako $f(x+T) = f(x)$.
Periodične su samo trigonometrijske f-je

F-ja nije periodična

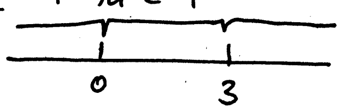
nule, presjek sa Y-osom, znak f-je

tačka oblika $(A, 0)$ je nula f-je, a tačka oblika $(0, B)$ je tačka presjeka sa Y-osom.

$f(x) = \dfrac{x}{x-3}$, $f(0) = \dfrac{0}{-3} = 0$

$(0,0)$ je nula f-je i presjek sa Y-osom

prekidi f-je i nule Y



x	$(-\infty, 0)$	$(0, 3)$	$(3, +\infty)$
x	−	+	+
x−3	−	−	+
Y	+	−	+

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

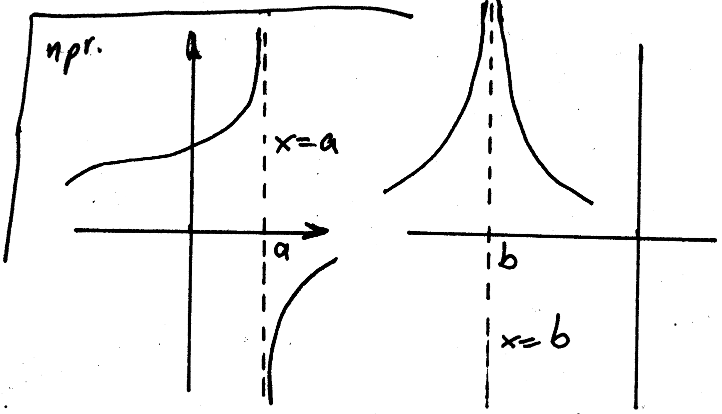
Neka je a tačka u kojoj f-ja nije definisana.

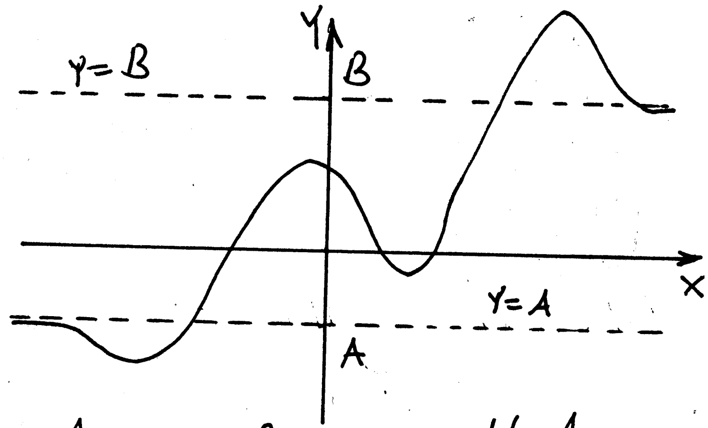
$\lim\limits_{x \to a-0} f(x) = -\infty$ (ili $+\infty$) $\Rightarrow$ $x = a$ je vertikalna asimptota

$\lim\limits_{x \to a+0} f(x) = +\infty$ (ili $-\infty$) $\Rightarrow$ $x = a$ je vertikalna asimptota

$\lim\limits_{x \to \infty} f(x) = A$, $A \neq +\infty$ i $A \neq -\infty$ $\Rightarrow$ $Y = A$ je horizontalna asimptota

$\lim\limits_{x \to -\infty} f(x) = B$, $B \neq +\infty$ i $B \neq -\infty$ $\Rightarrow$ $Y = B$ je horizontalna asimptota



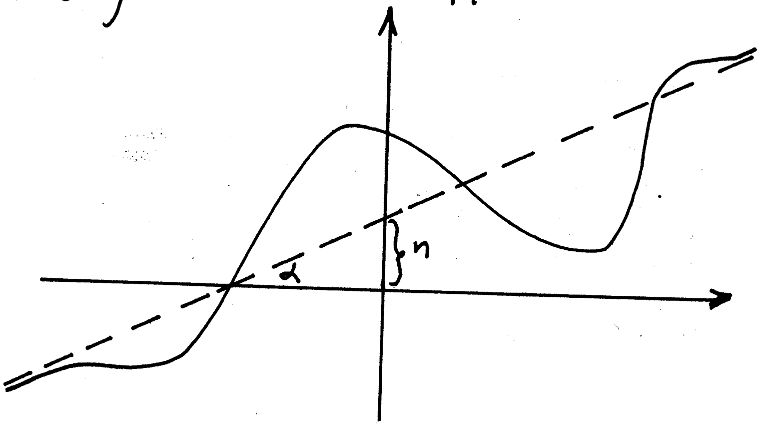


$x=a$ i $x=b$ su $V_o A_o$ $\qquad$ $y=A$ i $y=B$ su $H_o A_o$

Ako f-ja nema horizontalnu asimptotu onda tražimo kosu asimptotu u obliku $y=kx+n$.

$$k=\lim_{x\to\infty}\frac{f(x)}{x}, \quad n=\lim_{x\to\infty}[f(x)-kx]$$

Ako je $k=\pm\infty$ ili $k=0$ f-ja nema kosu asimptotu.



U beskonačnosti f-ja ne dodiruje asimptotu ali je u "normalnom" položaju u nekoj tački može sjeći.

Za $x=3$ f-ja nije definisana

$$\lim_{x\to 3-0}\frac{x}{x-3}=\frac{3-0}{3-0-3}=\frac{3-0}{-0}=-\infty$$

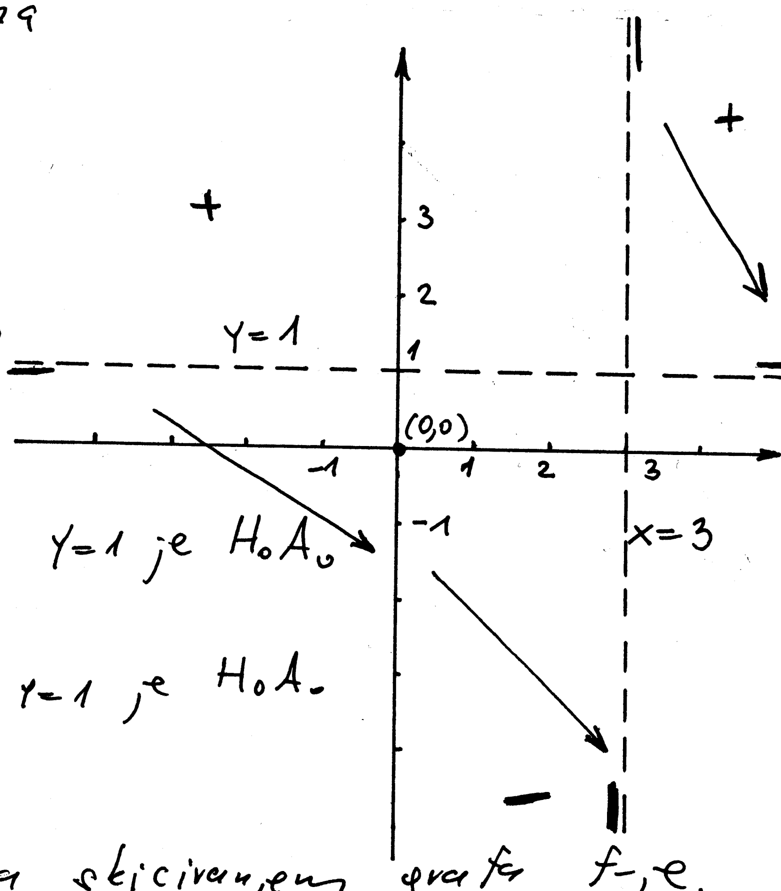
$\Rightarrow$ $x=3$ je $V_o A_o$ (sa lijeve str.)

$$\lim_{x\to 3+0}\frac{x}{x-3}=\frac{3+0}{3+0-3}=\frac{3+0}{+0}=+\infty$$

$\Rightarrow$ $x=3$ je $V_o A_o$ (sa desne str.)

$$\lim_{x\to+\infty}\frac{x\ /:x}{x-3\ /:x}=\lim_{x\to+\infty}\frac{1}{1-\underbrace{\frac{3}{x}}_{\to 0}}=1 \Rightarrow y=1 \text{ je } H_o A_o$$

$$\lim_{x\to-\infty}\frac{x}{x-3}=\lim_{x\to-\infty}\frac{1}{1-\frac{3}{x}}=1 \Rightarrow y=1 \text{ je } H_o A_o$$



F-ja nema kosu asimptotu.

Poslije ovog koraka počinjemo sa skiciranjem grafa f-je.

intervali rasta i opadanja

$$Y' = \left(\frac{x}{x-3}\right)' = \frac{1\cdot(x-3) - x\cdot 1}{x-3} = \frac{-3}{(x-3)^2} < 0 \quad \forall x \in D$$

f-ja Y $\searrow$ za $\forall (x \in D)$

ekstremi f-je

$$Y' = 0, \quad Y' = \frac{-3}{(x-3)^2} \neq 0 \quad \forall x \in D \Rightarrow$$ f-ja nema ekstrema

prevojne tačke i intervali konveksnosti i konkavnosti

Konveksnost ($\cup$) i konkavnost ($\cap$) f-je određujemo na osnovu znaka f-je Y''.
Ako je $\forall x \in (a,b)$ $Y''(x) < 0 \Rightarrow$ f-ja Y je $\cap$ na (a,b)
Ako je $\forall x \in (a,b)$ $Y''(x) > 0 \Rightarrow$ f-ja Y je $\cup$ na (a,b)
Za $Y''=0$ dobijemo tačke $x_1, x_2, \dots, x_n$ koje konkurišu za prevojne tačke. Tačka x_1 je prevojna tačka ako u njoj f-ja Y prelazi iz $\cup$ u $\cap$ i obrnuto

$$Y'' = \left(\frac{-3}{(x-3)^2}\right)' = \left(-3(x-3)^{-2}\right)' = 6(x-3)^{-3}\cdot 1 = \frac{6}{(x-3)^3} \neq 0 \Rightarrow$$ f-ja nema prevojnih tački

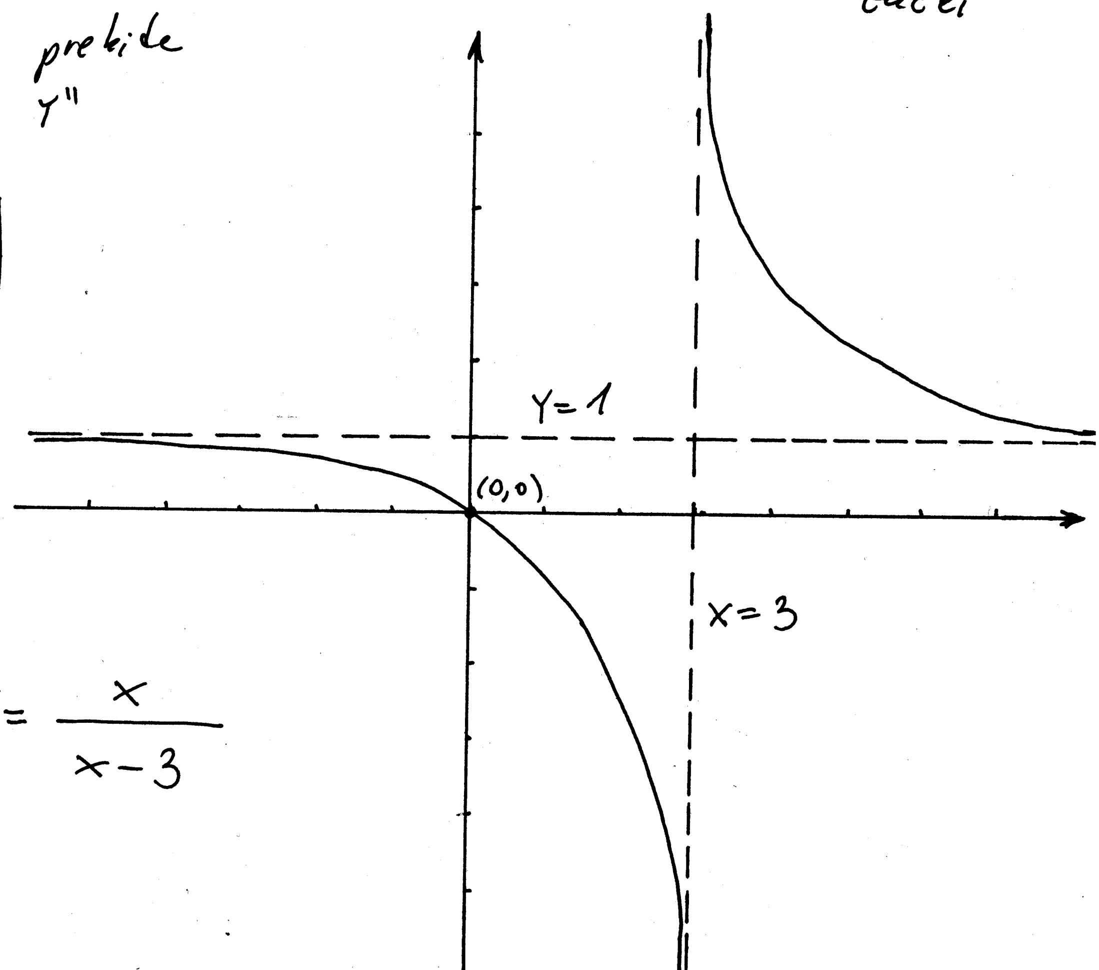
u tabelu stavljamo prekide f-je Y + nule f-je Y''

x	$(-\infty, 3)$	$(3, +\infty)$
Y''	$-$	$+$
Y	$\cap$	$\cup$

konveksnost i konkavnost

grafik f-je

$$Y = \frac{x}{x-3}$$



Ispitati f-ju i nacrtati joj grafik $y=\dfrac{3x}{1+x^3}$.

Rj. definiciono područje

$$1+x^3 \neq 0$$
$$x^3 \neq -1$$
$$x \neq -1$$
$$D: x \in (-\infty,-1)\cup(-1,+\infty)$$

parnost, neparnost, periodičnost

$$f(-x)=\frac{3\cdot(-x)}{1+(-x^3)}=-\frac{3x}{1-x^3}$$

f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa y-osom, znak f-je

$$y=0$$
$$\frac{3x}{1+x^3}=0$$
$$x=0$$

$(0,0)$ je nula f-je i presjek sa y-osom

x	$(-\infty,-1)$	$(-1,0)$	$(0,+\infty)$
$3x$	$-$	$-$	$+$
$1+x^3$	$-$	$+$	$+$
y	$+$	$-$	$+$

znak f-je

$-1 \quad 0 \quad 1$

ponašanje na krajevima intervala definisanosti i asimptote

za vrijednost $x=-1$ f-ja ima prekid

$$\lim_{x\to-1-0} f(x)=\lim_{x\to-1-0}\frac{3x}{1+x^3}=\frac{3(-1-0)}{1+(-1-0)^3}=\frac{3(-1-0)}{1-1-0}=\frac{-3-0}{-0}=+\infty \Rightarrow x=-1 \text{ je V.A.}$$

$$\lim_{x\to-1+0} f(x)=\lim_{x\to-1+0}\frac{3x}{1+x^3}=\frac{3(-1+0)}{1+(-1+0)^3}=\frac{-3+0}{1-1+0}=\frac{-3+0}{+0}=-\infty \Rightarrow x=-1 \text{ je V.A.}$$

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{3x \;:x}{1+x^3\;:x}=\lim_{x\to-\infty}\frac{3}{\underbrace{\frac{1}{x}}_{\to 0}+x^2}=0 \Rightarrow y=0 \text{ je H.A.}$$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{3}{\underbrace{\frac{1}{x}}_{\to 0}+x^2}=0 \Rightarrow y=0 \text{ je H.A.}$$

f-ja nema K.A.

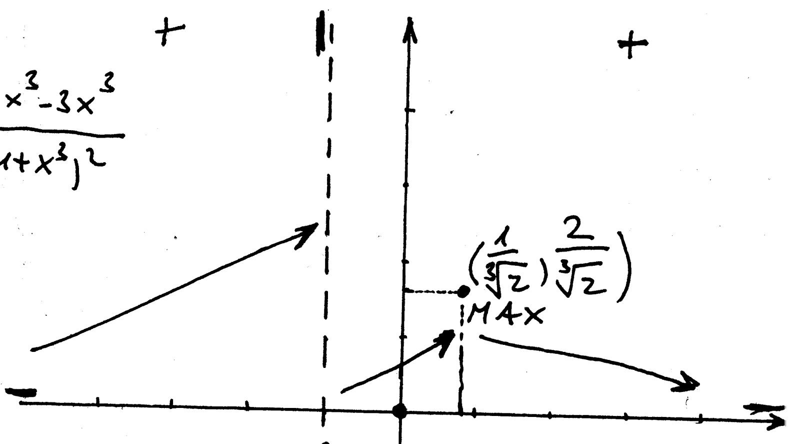
rast i opadanje

$$y'=\left(\frac{3x}{1+x^3}\right)'=3\cdot\frac{1\cdot(1+x^3)-x\cdot 3x^2}{(1+x^3)^2}=3\,\frac{1+x^3-3x^3}{(1+x^3)^2}$$

$$y'=3\cdot\frac{1-2x^3}{(1+x^3)^2}$$

$y'=0$ akko $1-2x^3=0$

$$2x^3=1$$
$$x^3=\frac{1}{2}$$
$$x=\frac{1}{\sqrt[3]{2}}\approx 0{,}8$$



x	$(-\infty,-1)$	$(-1,\frac{1}{\sqrt[3]{2}})$	$(\frac{1}{\sqrt[3]{2}},+\infty)$
y'	$+$	$+$	$-$
y	↗	↗	↘

MAX

$-1 \quad \frac{1}{\sqrt[3]{2}}$

prekidi y + nule y'

ekstremi f-je

Na osnovu tabele

$$f\left(\frac{1}{\sqrt[3]{2}}\right)=\frac{\frac{3}{\sqrt[3]{2}}}{1+\frac{1}{2}}=\frac{\frac{3}{\sqrt[3]{2}}}{\frac{3}{2}}=\frac{2}{\sqrt[3]{2}}\approx 1{,}6$$

$\left(\frac{1}{\sqrt[3]{2}},\frac{2}{\sqrt[3]{2}}\right)$ je tačka maksimuma prevojne tačke i intervali konveksnosti i konkavnosti

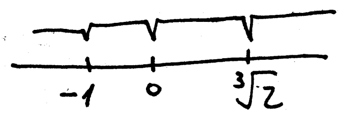
$$y' = 3 \cdot \frac{1-2x^3}{(1+x^3)^2}, \quad y'' = 3 \cdot \frac{-6x^2 \cdot (1+x^3)^{\cancel{2}} - (1-2x^3) \cdot 2\cancel{(1+x^3)} \cdot 3x^2}{(1+x^3)^3 \cdot \cancel{(1+x^3)}} =$$

$$= 3 \cdot \frac{-6x^2 - 6x^5 - 6x^2 + 12x^5}{(1+x^3)^3} = 3 \cdot \frac{6x^5 - 12x^2}{(1+x^3)^3}$$

$$y'' = 18 \cdot \frac{x^5 - 2x^2}{(1+x^3)^3} = \frac{18x^2(x^3-2)}{(1+x^3)^3}$$

$y''=0$ akko $x=0$ ili $x^3-2=0$

$x_1=0$ $\qquad x_2=\sqrt[3]{2} \approx 1{,}3$



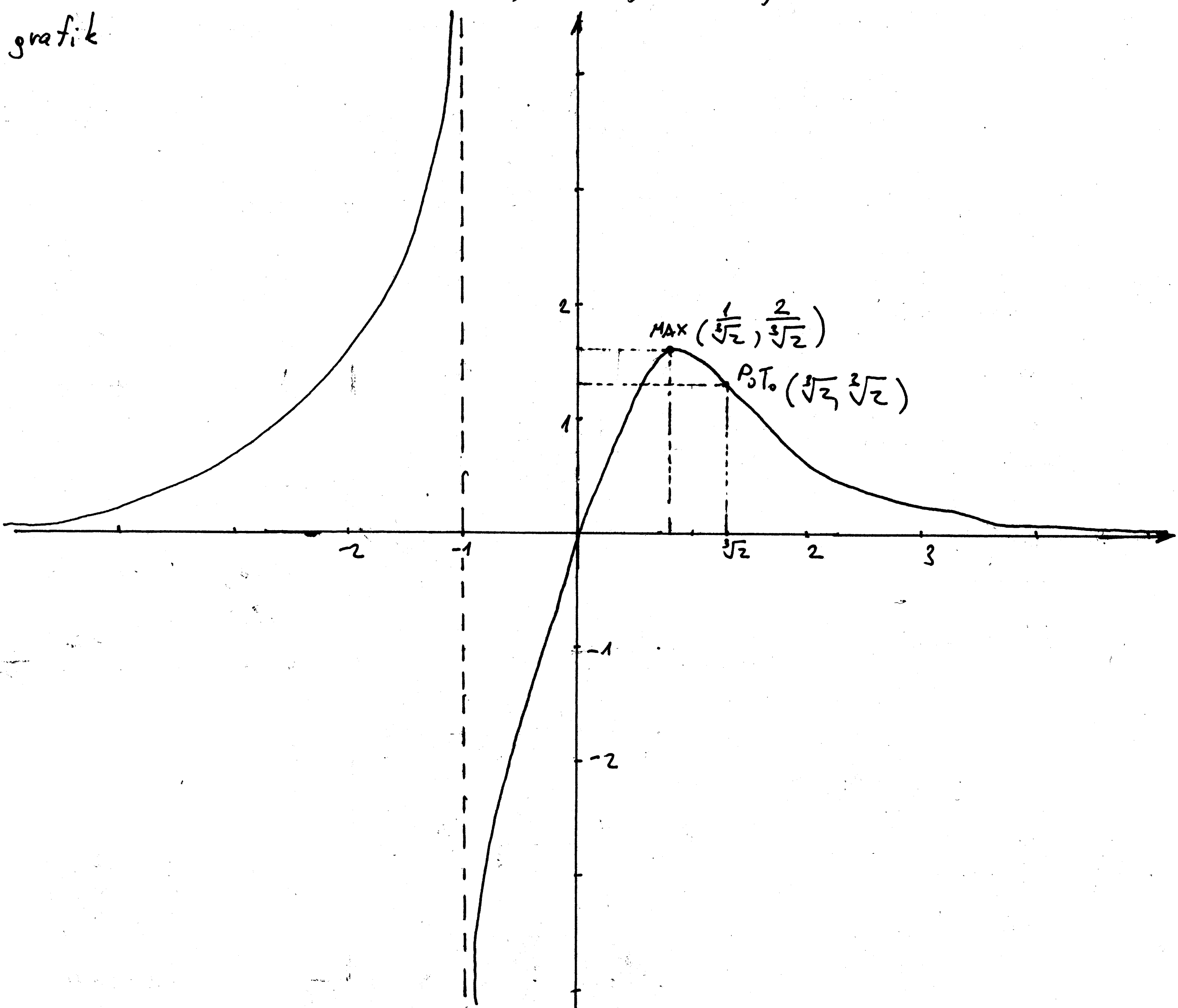
x	$(-\infty,-1)$	$(-1,0)$	$(0,\sqrt[3]{2})$	$(\sqrt[3]{2},+\infty)$
y''	+	−	−	+
y	∪	∩	∩	∪

PoTo

$$f(\sqrt[3]{2}) = \frac{3\sqrt[3]{2}}{1+2} = \sqrt[3]{2}$$

$(\sqrt[3]{2}, \sqrt[3]{2})$ je prevojna tačka

grafik



(#) Ispitati f-ju i nacrtati joj grafik $Y=\frac{(2x-1)^3}{(x+2)^2}$.

Rj.

definiciono područje
$D: x\in\mathbb{R}\setminus\{-2\}$

parnost, neparnost, periodičnost
D nije simetrično $\Rightarrow$ f-ja nije ni parna ni neparna
f-ja nije periodična

nule, presjek sa y-osom, znak f-je

$Y=0$ akko $(2x-1)^3=0$
$2x-1=0$
$x=\frac{1}{2}$

$(\frac{1}{2},0)$ je nula f-je

$f(0)=\frac{(-1)^3}{2^2}=-\frac{1}{4}$

$(0,-\frac{1}{4})$ je tačka presjeka sa y-osom

x	$(-\infty,-2)$	$(-2,\frac{1}{2})$	$(\frac{1}{2},+\infty)$
$(2x-1)^3$	$-$	$-$	$+$
Y	$-$	$-$	$+$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

za $x=-2$ f-ja ima prekid

$\lim\limits_{x\to-2-0} f(x)=\lim\limits_{x\to-2-0}\frac{(2x-1)^3}{(x+2)^2}=\frac{(2\cdot(-2-0)-1)^3}{(-2-0+2)^2}=\frac{(-5-0)^3}{+0}=-\infty \Rightarrow x=-2$ je V.A. (sa lijeve strane)

$\lim\limits_{x\to-2+0} f(x)=\lim\limits_{x\to-2+0}\frac{(2x-1)^3}{(x+2)^2}=\frac{(2\cdot(-2+0)-1)^3}{(-2+0+2)^2}=\frac{(-5-0)^3}{+0}=-\infty \Rightarrow x=-2$ je V.A. (sa desne strane)

$(2x-1)^3=(2x)^3+3\cdot(2x)^2\cdot(-1)+3\cdot 2x\cdot(-1)^2+(-1)^3=8x^3-12x^2+6x-1$

1
1 1
1 2 1
1 3 3 1

$\lim\limits_{x\to-\infty} f(x)=\lim\limits_{x\to-\infty}\frac{(2x-1)^3}{(x+2)^2}=\lim\limits_{x\to-\infty}\frac{8x^3-12x^2+6x-1 \quad /:x^3}{x^2+4x+2 \quad /:x^3}=\lim\limits_{x\to-\infty}\frac{8-\frac{12}{x}+\frac{6}{x^2}-\frac{1}{x^3}}{\frac{1}{x}+\frac{4}{x^2}+\frac{2}{x^3}}=-\infty$

$\lim\limits_{x\to+\infty} f(x)=\lim\limits_{x\to+\infty}\frac{8x^3-12x^2+6x-1 \quad /:x^2}{x^2+4x+2 \quad /:x^2}=\lim\limits_{x\to+\infty}\frac{8x-12+\frac{6}{x}-\frac{1}{x^2}}{1+\frac{4}{x}+\frac{2}{x^2}}=+\infty$

f-ja nema H.A.

kosa asimptota je oblika $y=kx+n$

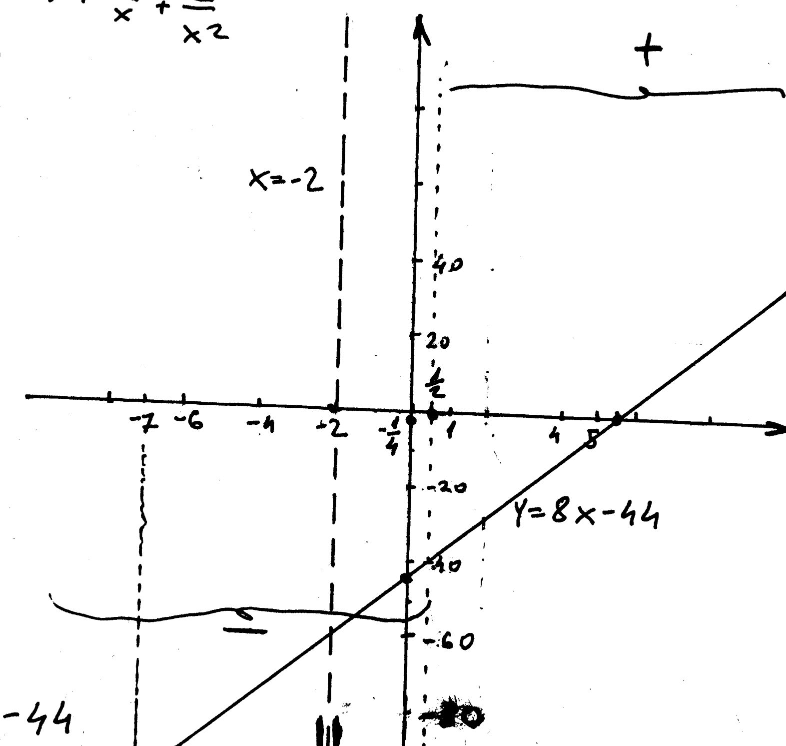
$k=\lim\limits_{x\to\infty}\frac{f(x)}{x}=\lim\limits_{x\to\infty}\frac{8x^3-12x^2+6x-1 \quad /:x^3}{x^3+4x^2+2x \quad /:x^3}$
$=\lim\limits_{x\to\infty}\frac{8-\frac{12}{x}+\frac{6}{x^2}-\frac{1}{x^3}}{1+\frac{4}{x}+\frac{2}{x^2}}=8$

$n=\lim\limits_{x\to\infty}[f(x)-k\cdot x]=\lim\limits_{x\to\infty}\left(\frac{(2x-1)^3}{(x+2)^2}-8x\right)=$

$=\lim\limits_{x\to\infty}\frac{8x^3-12x^2+6x-1-8x(x^2+4x+2)}{(x+2)^2}=$

$=\lim\limits_{x\to\infty}\frac{\cancel{8x^3}-12x^2+6x-1-\cancel{8x^3}-32x^2-16x}{x^2+4x+4}$

$=\lim\limits_{x\to\infty}\frac{-44x^2-10x-1 \quad :x^2}{x^2+4x+4 \quad :x^2}=\lim\limits_{x\to\infty}\frac{-44-\frac{10}{x}-\frac{1}{x^2}}{1+\frac{4}{x}+\frac{4}{x^2}}=-44$



$y = 8x - 44$ je K. A. (počinjemo sa skiciravanjem grafa)

$\left(y = 8x-44, \quad y = 0 \Rightarrow 8x = 44, \quad x = \frac{44}{8} = \frac{11}{2} = 5{,}5 \qquad x = 0 \Rightarrow y = -44 \right)$

rast i opadanje

$$y' = \left(\frac{(2x-1)^3}{(x+2)^2}\right)' = \frac{3(2x-1)^2\cdot 2(x+2)^{\cancel{2}\,1} - (2x-1)^3\,\cancel{2(x+2)}}{(x+2)^{\cancel{4}\,3}} = \frac{2(2x-1)^2(3x+6-2x+1)}{(x+2)^3} = \frac{2(2x-1)^2(x+7)}{(x+2)^3}$$

$y' = 0$ akko $x = \frac{1}{2}$ i $x = -7$

$-7 \quad -2 \quad \frac{1}{2}$ ← nule y' + prekidi od y

x	$(-\infty,-7)$	$(-7,-2)$	$(-2,\frac{1}{2})$	$(\frac{1}{2},+\infty)$
y'	+	−	+	+
y	↗	↘	↗	↗

max

rast i opadanje

$$f(-7) = \frac{(-15)^3}{(-5)^2} = \frac{-3375}{25} = -135$$

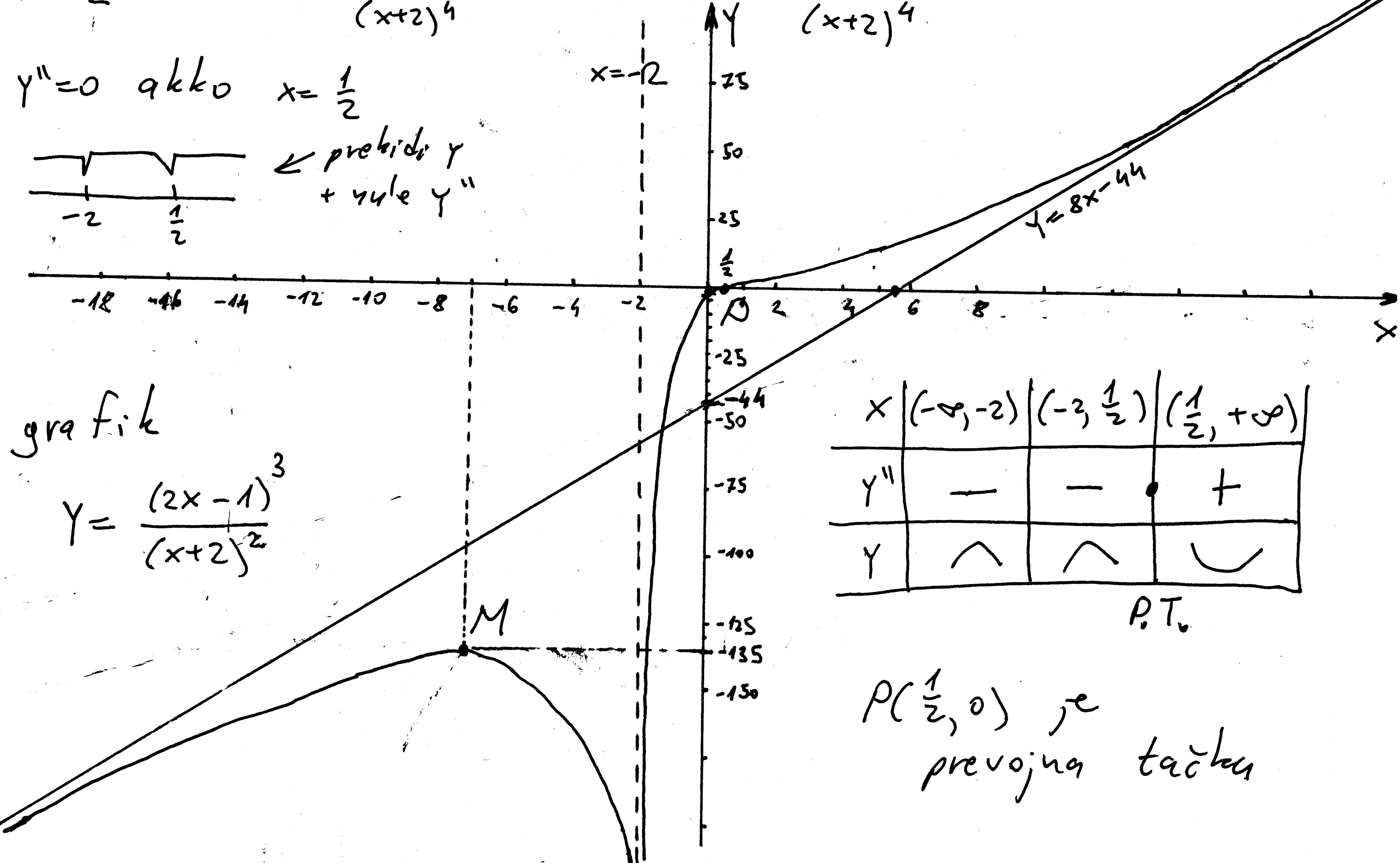
ekstremi f-je: Na osnovu tabele rasta i opadanja $M(-7,-135)$ je tačka maksimuma

prevojne tačke i intervali konveksnosti i konkavnosti

$$y'' = \left(2\,\frac{(2x-1)^2(x+7)}{(x+2)^3}\right)' = 2\cdot\frac{\left[2(2x-1)\cdot 2\cdot(x+7) + (2x-1)^2\right](x+2)^{\cancel{3}\,1} - (2x-1)^2(x+7)\,3\,\cancel{(x+2)^2}}{(x+2)^{\cancel{6}\,4}} =$$

$$= 2\cdot\frac{\left[(2x-1)(4x+28+2x-1)\right](x+2) - 3(2x-1)^2(x+7)}{(x+2)^4} = 2\cdot\frac{(2x-1)\left[\overbrace{(6x+27)(x+2)}^{6x^2+12x+27x+54} - 3\overbrace{(2x-1)(x+7)}^{2x^2+13x-7}\right]}{(x+2)^4}$$

$$y'' = 2\cdot\frac{(2x-1)(\cancel{6x^2}+39x+54-\cancel{6x^2}-39x+21)}{(x+2)^4} = \frac{150(2x-1)}{(x+2)^4}$$



$y'' = 0$ akko $x = \frac{1}{2}$

$-2 \quad \frac{1}{2}$ ← prekidi y + nule y''

grafik

$$y = \frac{(2x-1)^3}{(x+2)^2}$$

x	$(-\infty,-2)$	$(-2,\frac{1}{2})$	$(\frac{1}{2},+\infty)$
y''	−	−	+
y	∩	∩	∪

P.T.

$P(\frac{1}{2}, 0)$ je prevojna tačka

\# Ispitati i grafički predstaviti f-ju $Y = \dfrac{x^2+5x}{x^2+2x+1}$.

Rj:

definiciono područje

$x^2+2x+1 \neq 0$

$D = 4-4 = 0$

$(x+1)^2 \neq 0$

$x \neq -1$

$D: x \in \mathbb{R} \setminus \{-1\}$

parnost, neparnost, periodičnost

D nije simetrično $\Rightarrow$ f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa y-osom, znak f-je

$Y=0$ akko $x^2+5x=0$

$x(x+5)=0$

$x_1 = 0$ ili $x_2 = -5$

$(0,0)$ i $(-5,0)$ su nule f-je

$(0,0)$ je tačka presjeka sa Y-osom.

$Y = \dfrac{x(x+5)}{(x+1)^2}$ $\quad -5 \quad -1 \quad 0$

x	$(-\infty,-5)$	$(-5,-1)$	$(-1,0)$	$(0,+\infty)$
x	$-$	$-$	$-$	$+$
x+5	$-$	$+$	$+$	$+$
Y	$+$	$-$	$-$	$+$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

za $x=-1$ f-ja ima prekid.

$$\lim_{x\to-1-0} f(x) = \lim_{x\to-1-0} \frac{x(x+5)}{(x+1)^2} = \frac{(-1-0)(-1-0+5)}{(-1-0+1)^2} = \frac{(-1-0)(4-0)}{+0} = -\infty \Rightarrow x=-1 \text{ je } V_oA_o$$

$$\lim_{x\to-1+0} f(x) = \lim_{x\to-1+0} \frac{x(x+5)}{(x+1)^2} = \frac{(-1+0)(-1+0+5)}{(-1+0+1)^2} = \frac{(-1+0)(4+0)}{+0} = -\infty \Rightarrow x=-1 \text{ je } V_oA_o$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x^2+5x \;/:x^2}{x^2+2x+1 \;/:x^2} = \lim_{x\to-\infty} \frac{1+\frac{5}{x}}{1+\frac{2}{x}+\frac{1}{x^2}} = 1 \Rightarrow Y=1 \text{ je } H_oA_o$$

isto vrijedi i za $\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \dfrac{1+\frac{5}{x}}{1+\frac{2}{x}+\frac{1}{x^2}} = 1 \Rightarrow Y=1$ je H_oA_o

nakon ovog koraka počinjemo skicirati graf

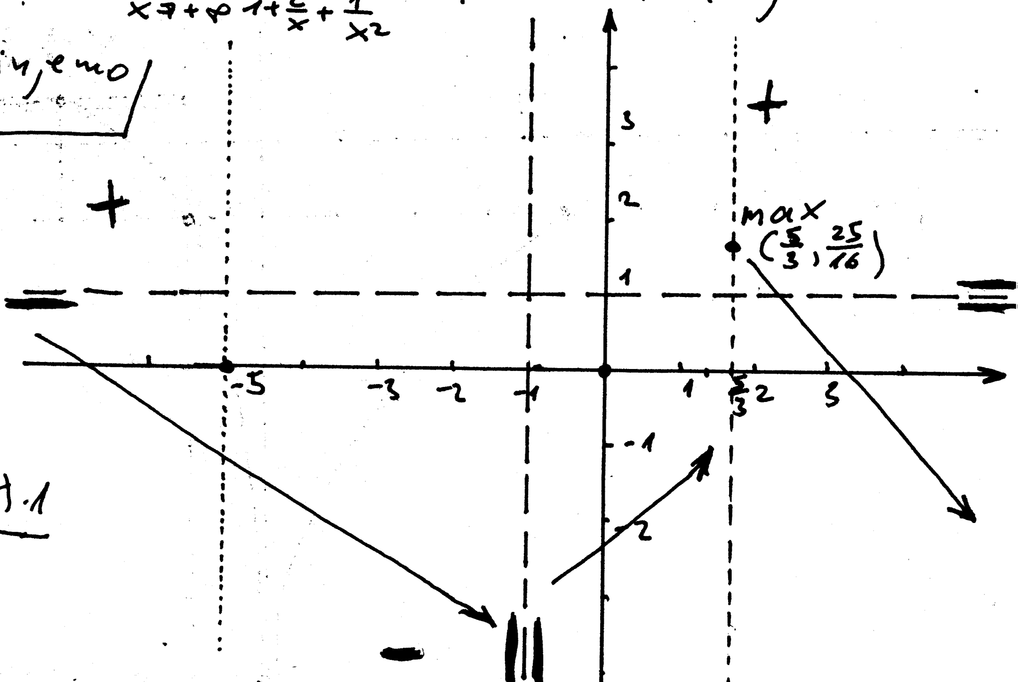
f-ja nema K_oA_o

rast i opadanje

$$Y' = \left(\frac{x^2+5x}{(x+1)^2}\right)' =$$

$$= \frac{(2x+5)(x+1)^{\cancel{2}\,1} - (x^2+5x)\,\cancel{2(x+1)}\cdot 1}{(x+1)^{\cancel{4}\,3}}$$

$$= \frac{\cancel{2x^2}+5x+2x+5-\cancel{2x^2}-10x}{(x+1)^3}$$



$$Y' = \frac{-3x+5}{(x+1)^3}$$

← nule Y' + prekidi Y

$-1 \qquad \frac{5}{3}$

$Y'=0$ akko $-3x+5=0$

$-3x=-5$

$x=\frac{5}{3} \approx 1,6667$

$Y'(-2) = \frac{11}{-1} < 0$

X	$(-\infty,-1)$	$(-1,\frac{5}{3})$	$(\frac{5}{3},+\infty)$
Y'	$-$	$+$	$-$
Y	↘	↗	↘

max

rast i opadanje

ekstremi f-je

na osnovu tabele rasta i opadanja f-ja ima maksimum za $x=\frac{5}{3}$

$$f\left(\frac{5}{3}\right) = \frac{\frac{25}{9}+5\cdot\frac{5}{3}}{\left(\frac{5}{3}+1\right)^2} = \frac{\frac{25+25\cdot 3}{9}}{\left(\frac{8}{3}\right)^2} = \frac{\frac{100}{\cancel{9}}:2}{\frac{64}{\cancel{9}}:2} = \frac{50}{32} = \frac{25}{16} \approx 1,5625$$

$M\left(\frac{5}{3},\frac{25}{16}\right)$ je tačka maksimuma

prevojne tačke i intervali konveksnosti i konkavnosti

$$Y'' = \left(\frac{-3x+5}{(x+1)^3}\right)' = \frac{-3(x+1)^{\cancel{3}\,1} - (-3x+5)3\cancel{(x+1)^2}\cdot 1}{(x+1)^4\cdot\cancel{(x+1)^2}} = \frac{-3x-3+9x-15}{(x+1)^4} = \frac{6x-18}{(x+1)^4}$$

$Y'' = 6\cdot\frac{x-3}{(x+1)^4}$, $Y''=0$ akko $x=3$

← prekidi Y + nule Y''

$-1 \qquad 3$

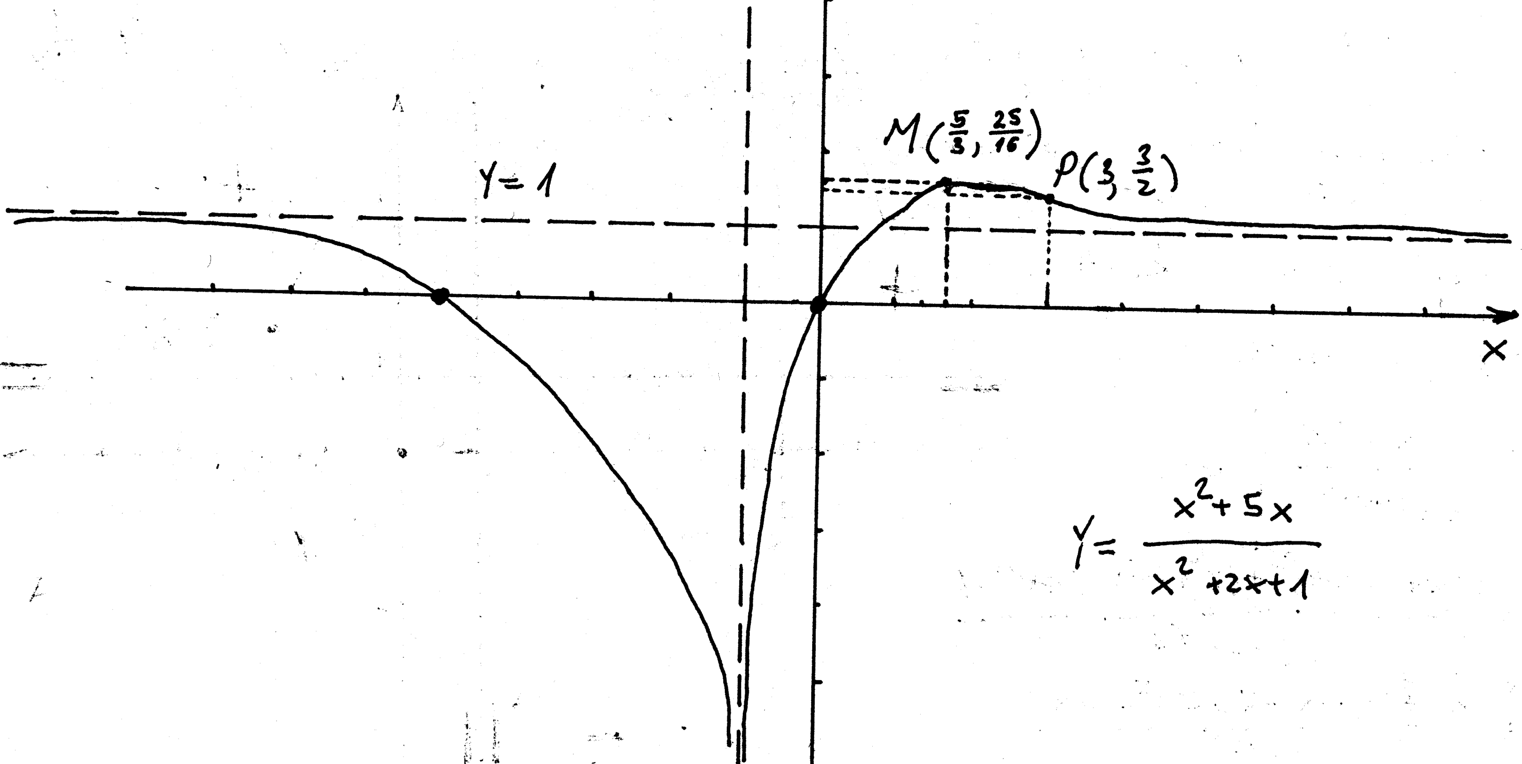
X	$(-\infty,-1)$	$(-1,3)$	$(3,+\infty)$
Y''	$-$	$-$	$+$
Y	⌒	⌒	∪

P_0T_0

$$f(3) = \frac{3^2+5\cdot 3}{(3+1)^2} = \frac{9+15}{16} = \frac{24:4}{16:4} = \frac{6:2}{4:2} = \frac{3}{2} = 1,5$$

$P\left(3,\frac{3}{2}\right)$ je prevojna tačka

grafik



(#) Odrediti parametre a i b tako da f-ja $y=\frac{x}{x^2+ax+b}$ ima ekstrem u tački $T(2,\frac{1}{7})$. Zatim ispitati tako dobijenu f-ju i nacrtati joj grafik.

Rj:

$f(2)=\frac{1}{7}$

$$\frac{2}{4+2a+b}=\frac{1}{7}$$

$4+2a+b=14$

$2a+b=10$

Kandidat za ekstreme su stacionarne tačke

$$y'=\frac{x^2+ax+b-x(2x+a)}{(x^2+ax+b)^2}=\frac{x^2+\cancel{ax}+b-2x^2-\cancel{ax}}{(x^2+ax+b)^2}$$

$$y'=\frac{-x^2+b}{(x^2+ax+b)^2}$$

Potreban uslov da f-ja y ima ekstrem u tački $T(2,\frac{1}{7})$ je $y'(2)=0$.

$-4+b=0$

$b=4$

$2a+4=10$

$2a=6$

$a=3$

$$y=\frac{x}{x^2+3x+4}$$

definiciono područje

$x^2+3x+4\neq 0$

$D=9-16<0$

$a>0 \quad x^2+3x+4>0 \quad \forall x\in\mathbb{R}$

$\mathcal{D}: x\in\mathbb{R}$

parnost, neparnost, periodičnost

$$f(-x)=\frac{-x}{x^2-3x+4}$$ f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa y-osom, znak

$f(x)=0$ akko $x=0$

$(0,0)$ je nula f-je i presjek sa y-osom

x	$(-\infty,0)$	$(0,+\infty)$
y	$-$	$+$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

f-ja nema prekida $\Rightarrow$ f-ja nema V. A.

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{x}{x^2+3x+4}\begin{matrix}:x\\:x\end{matrix}=\lim_{x\to\infty}\frac{1}{x+3+\frac{4}{x}}=\frac{1}{\infty}=0$$

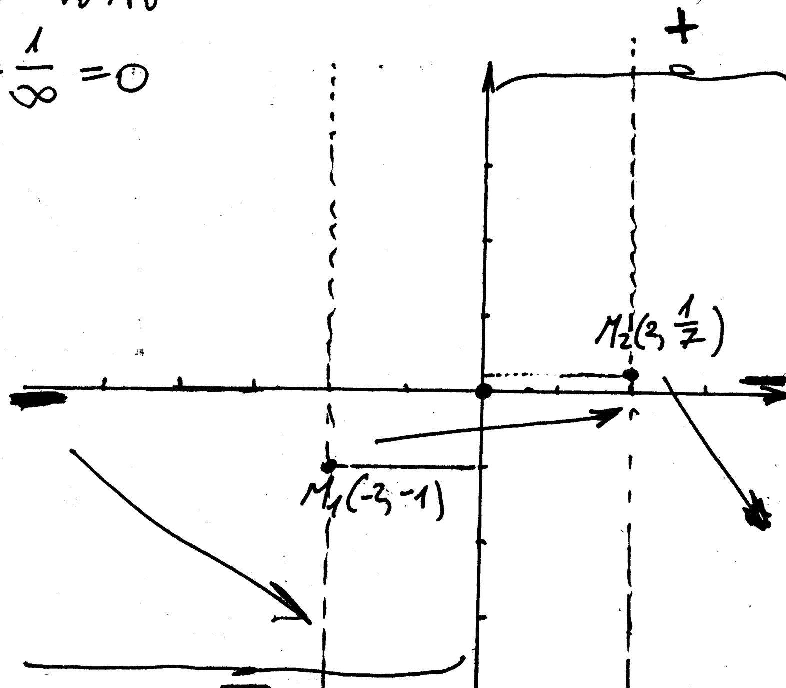
$\Rightarrow y=0$ je H. A.

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{1}{x+3+\frac{4}{x}}=\frac{1}{-\infty}=0$$

$\Rightarrow y=0$ je H. A.

F-ja nema K. A.

Poslije ovog koraka počinjemo skicirati grafik.



rast i opadanje

$$y' = \frac{-x^2+b}{(x^2+ax+b)^2} \Rightarrow y' = \frac{4-x^2}{(x^2+3x+4)^2}$$

$y' = 0$ akko $4-x^2=0$

$x_1=-2,\ x_2=2$

$\frac{-2}{8-6}$

$\frac{2}{8+6}=\frac{2}{14}$

x	$(-\infty,-2)$	$(-2,2)$	$(2,+\infty)$
y'	$-$	$+$	$-$
y	$\searrow$	$\nearrow$	$\searrow$

rast i opadanje

min $f(-2)=-1$ max $f(2)=\frac{1}{7}$

ekstremi f-je

Na osnovu tabele $M_1(-2,-1)$ je tačka min a $M_2(2,\frac{1}{7})$ je max.

prevojne tačke i intervali konv. i konk.

$$y'' = \left(\frac{4-x^2}{(x^2+3x+4)^2}\right)' =$$

$$= \frac{-2x\cancel{(x^2+3x+4)^2} - (4-x^2)2\cancel{(x^2+3x+4)}\cdot(2x+3)}{(x^2+3x+4)^{\cancel{4}\,3}} = \frac{-2\left[\cancel{x^3}+\cancel{3x^2}+4x+8x+12\cancel{-2x^3}\cancel{-3x^2}\right]}{(x^2+3x+4)^3}$$

$$y'' = -2\cdot\frac{-x^3+12x+12}{(x^2+3x+4)^3} = 2\,\frac{x^3-12x-12}{(x^2+3x+4)^3}$$

$y''=0$ akko $x^3-12x-12=0$

$x_1 \approx 3{,}88 \quad x_2 \approx -1{,}11$

$x_3 \approx -2{,}77$

(vrijednosti x_1, x_2 i x_3 su nađene pomoću digitrona koji ima opciju da nađe nule polinoma)

$-2{,}77 \quad -1{,}11 \quad 3{,}88$

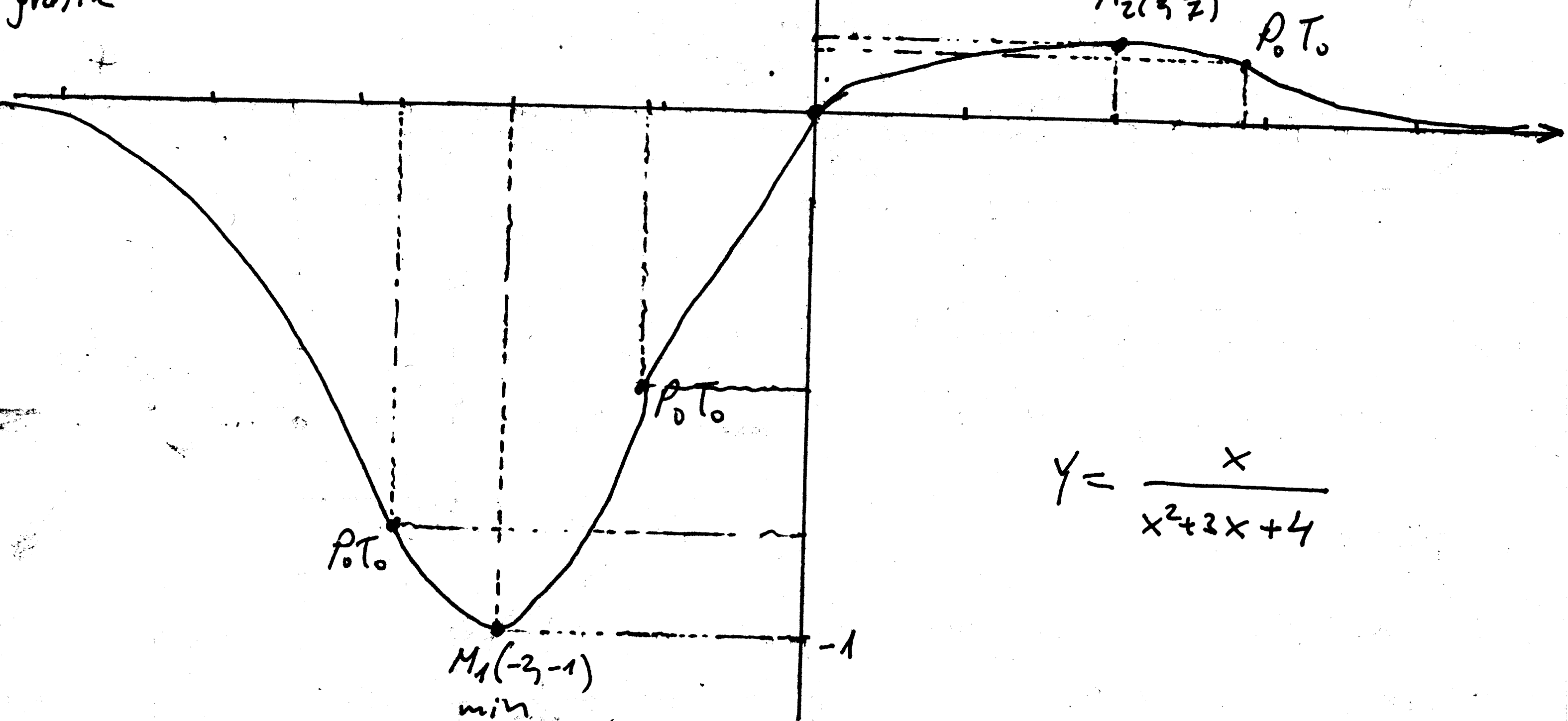
x	$(-\infty,-2{,}77)$	$(-2{,}77,-1{,}11)$	$(-1{,}11,\ 3{,}88)$	$(3{,}88,+\infty)$
y''	$-$	$+$	$-$	$+$
y	$\frown$	$\smile$	$\frown$	$\smile$

$P_0T_0 \quad P_0T_0 \quad P_0T_0$

$f(-2{,}77)\approx -0{,}82$ $\quad f(3{,}88)\approx 0{,}13$

$f(-1{,}11)\approx -0{,}58$

grafik



Ispitati i grafički predstaviti f-ju $y = x e^{\frac{1}{x}}$.

Rj: definiciono područje

$x \neq 0$, $\mathcal{D}: x \in \mathbb{R} \setminus \{0\}$

parnost, neparnost, periodičnost

$f(-x) = -x e^{\frac{1}{-x}} = -x e^{-\frac{1}{x}}$

f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa Y-osom, znak f-je

$x e^{\frac{1}{x}} = 0$

$x = 0$ ili $e^{\frac{1}{x}} = 0$

nije definisano $\quad e^{x} \neq 0 \ \forall x \in \mathbb{R}$

f-ja nema nulu

f(0) nije definisano

f-ja ne siječe Y-osu

$e^{\frac{1}{x}} > 0 \quad \forall x \in \mathcal{D}$

X	$(-\infty, 0)$	$(0, +\infty)$
Y	−	+

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

$x = 0$ f-ja ima prekid

$\left(-\frac{1}{x}\right)' = \left(-x^{-1}\right)'$

$$\lim_{x\to -0} f(x) = \lim_{x\to -0} x e^{\frac{1}{x}} = (-0)\cdot e^{\frac{1}{-0}} = (-0)\cdot e^{-\infty} = \frac{-0}{e^{\infty}} = \frac{-0}{\infty} = 0$$

$$\lim_{x\to +0} f(x) = \lim_{x\to +0} x e^{\frac{1}{x}} \left(= 0\cdot\infty\right) = \lim_{x\to +0} \frac{x}{e^{-\frac{1}{x}}} \left(= \frac{0}{0}\right) \overset{L.P.}{=\!=} \lim_{x\to +0} \frac{1}{e^{-\frac{1}{x}}\cdot\frac{1}{x^2}} = \lim_{x\to +0} \frac{x^2}{e^{-\frac{1}{x}}} = \ldots$$

pokušat ćemo na drugi način:

$$\lim_{x\to +0} f(x) = \lim_{x\to +0} x e^{\frac{1}{x}} \left(= 0\cdot\infty\right) = \lim_{x\to 0+} \frac{e^{\frac{1}{x}}}{x^{-1}} \left(= \frac{\infty}{\infty}\right) \overset{L.P.}{=\!=} \lim_{x\to 0+} \frac{e^{\frac{1}{x}}\cdot\left(\frac{1}{x}\right)'}{\left(\frac{1}{x}\right)'} = e^{\frac{1}{0+}} = \infty$$

$\Rightarrow x = 0$ je V.A.

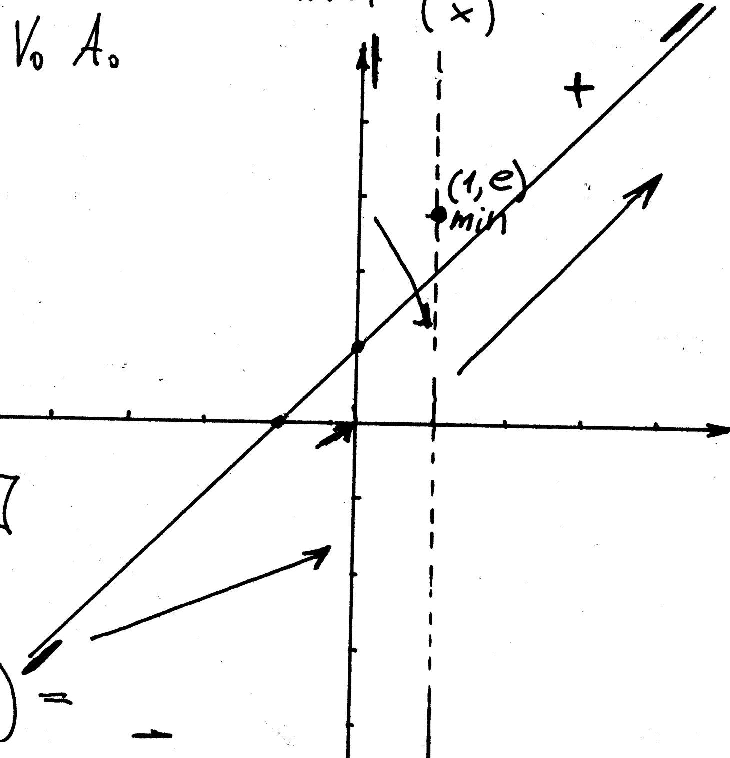
$$\left.\begin{aligned} \lim_{x\to -\infty} f(x) &= \lim_{x\to -\infty} x e^{\frac{1}{x}} = -\infty\cdot 1 = -\infty \\ \lim_{x\to +\infty} f(x) &= \lim_{x\to +\infty} x e^{\frac{1}{x}} = +\infty\cdot 1 = \infty \end{aligned}\right\} \Rightarrow$$

$\Rightarrow$ f-ja nema H.A.

$y = kx + n$, $\quad k = \lim_{x\to\infty} \frac{f(x)}{x}$, $\quad n = \lim_{x\to\infty} [f(x) - k\cdot x]$

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} e^{\frac{1}{x}} = e^{0} = 1$$

$$n = \lim_{x\to\infty} \left[x e^{\frac{1}{x}} - x\right] = \lim_{x\to\infty} x\left(e^{\frac{1}{x}} - 1\right) \left(= \infty\cdot 0\right) =$$



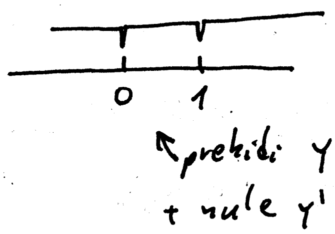
$$= \lim_{x\to\infty} \frac{e^{\frac{1}{x}}-1}{\frac{1}{x}} \left(= \frac{0}{0}\right) \overset{L.P.}{=} \lim_{x\to\infty} \frac{e^{\frac{1}{x}} \cdot \cancel{\left(\frac{1}{x}\right)'}}{\cancel{\left(\frac{1}{x}\right)'}} = \lim_{x\to\infty} e^{\frac{1}{x}} = e^0 = 1$$

$Y = x + 1$ je K. A.

rast i opadanje

$$Y' = \left(x e^{\frac{1}{x}}\right)' = e^{\frac{1}{x}} + x \cdot e^{\frac{1}{x}} \cdot (x^{-1})' = e^{\frac{1}{x}} + x e^{\frac{1}{x}} \cdot (-x^{-2}) = e^{\frac{1}{x}}\left(1 + x \cdot \left(-\frac{1}{x^2}\right)\right)$$

$$Y' = e^{\frac{1}{x}}\left(1 - \frac{1}{x}\right)$$



$Y' = 0$ akko $1 - \frac{1}{x} = 0$

$x = 1$

X	$(-\infty, 0)$	$(0, 1)$	$(1, +\infty)$
Y'	+	−	+
Y	↗	↘	↗

MIN

rast i opadanje

ekstremi f-je

na osnovu tabele rasta i opadanja f-ja ima minimum u tački $(1, f(1))$. $f(1) = 1 \cdot e^{\frac{1}{1}} = e$ $f_{min}(1) = e$ $(1, e)$

$e \approx 2{,}71$

prevojne tačke i intervali konveksnosti i konkavnosti

$$Y'' = \left(e^{\frac{1}{x}}\left(1 - \frac{1}{x}\right)\right)' = e^{\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)\left(1 - \frac{1}{x}\right) + e^{\frac{1}{x}} \cdot \left(-\left(-\frac{1}{x^2}\right)\right)$$

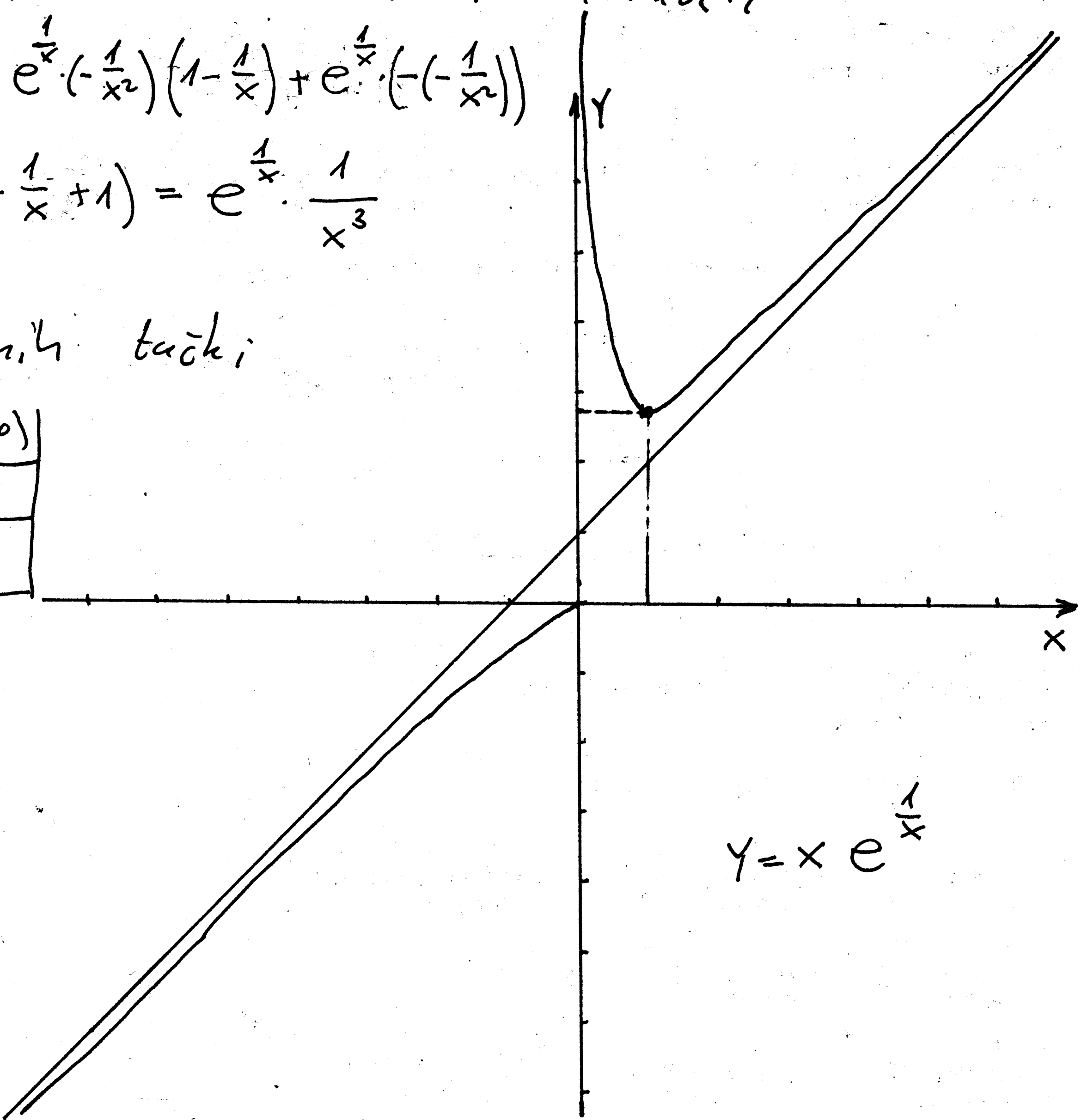
$$= e^{\frac{1}{x}} \cdot \frac{1}{x^2}\left(-1 + \frac{1}{x} + 1\right) = e^{\frac{1}{x}} \cdot \frac{1}{x^3}$$

$Y'' \neq 0 \quad \forall x \in \mathcal{D}$

nema prevojnih tački

X	$(-\infty, 0)$	$(0, +\infty)$
Y''	−	+
Y	∩	∪

grafik



(#) Ispitati f-ju i nacrtati joj grafik $y = x^3 e^{-\frac{x^2}{6}}$.

Rj.

definiciono područje

$\mathcal{D}: x \in \mathbb{R}$

parnost, neparnost, periodičnost

$y(-x) = (-x)^3 e^{-\frac{(-x)^2}{6}} = -x^3 e^{-\frac{x^2}{6}}$

f-ja je neparna (simetrična u odnosu na koordinatni početak). Dovoljno ju je ispitati za $x > 0$. F-ja nije periodična

nule, presjek sa y-osom, znak f-je

$x^3 \underbrace{e^{-\frac{x^2}{6}}}_{>0\ \forall x} = 0$

$x = 0$

(0,0) je nula f-je i presjek sa y-osom

x	$(-\infty, 0)$	$(0, +\infty)$
y	−	+

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

f-ja nema prekid $\Rightarrow$ nema V. A.

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} x^3 e^{-\frac{x^2}{6}} = \lim_{x\to+\infty} \frac{x^3}{e^{\frac{x^2}{6}}} \left(\frac{+\infty}{\infty}\right) \overset{L.P.}{=} \lim_{x\to+\infty} \frac{3x^2}{e^{\frac{x^2}{6}} \cdot \frac{1}{6} 2x} =$$

$$= \lim_{x\to+\infty} \frac{9x}{e^{\frac{x^2}{6}}} \left(\frac{\infty}{\infty}\right) \overset{L.P.}{=} \lim_{x\to\infty} \frac{9}{e^{\frac{x^2}{6}} \cdot \frac{1}{6} 2x} = \lim_{x\to\infty} \frac{27}{x e^{\frac{x^2}{6}}} = 0$$

$\Rightarrow$ $x = 0$ je H. A., F-ja nema K. A.

rast i opadanje

$$y' = 3x^2 e^{-\frac{x^2}{6}} + x^3 \cdot e^{-\frac{x^2}{6}} \cdot \left(-\frac{1}{6}\right) \cdot 2x$$

$$= 3x^2 e^{-\frac{x^2}{6}} - \frac{1}{3} x^4 e^{-\frac{x^2}{6}}$$

$$= x^2 e^{-\frac{x^2}{6}} \left(3 - \frac{1}{3}x^2\right) = x^2 e^{-\frac{x^2}{6}} \left(\frac{9 - x^2}{3}\right)$$

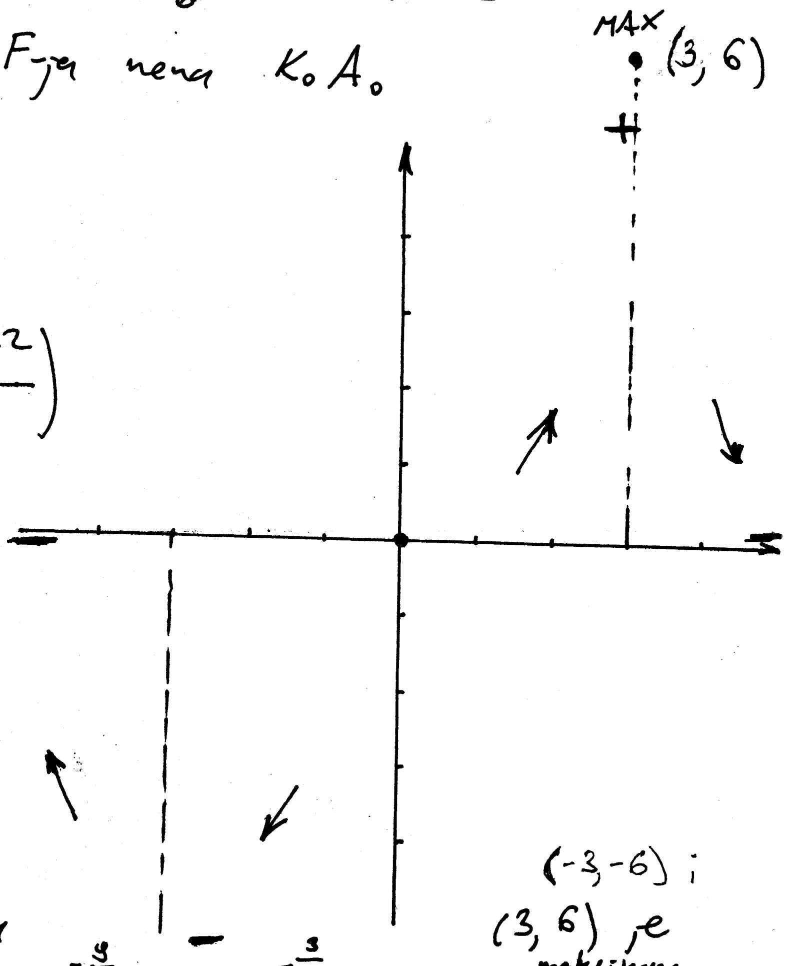
$y' = 0 \Leftrightarrow x_1 = 0,\ x_2 = -3,\ x_3 = 3$

← prekidi y + nule y'

x	$(0, 3)$	$(3, +\infty)$
y'	+	−
y	↗	↘

MAX

rast i opadanje



ekstremi f-je

Iz tabele rasta i opadanja vidimo da f-ja ima ekstrem za $x = 3$ $\quad f(3) = 27 e^{-\frac{9}{6}} = 27 e^{-\frac{3}{2}} \approx 6$

(-3, -6) i (3, 6) je maksimum f-je

prevojne tačke i intervali konveksnosti i konkavnosti

$$y''=\left(x^2 e^{-\frac{x^2}{6}}\frac{1}{3}(9-x^2)\right)' = 2x e^{-\frac{x^2}{6}}\cdot\frac{1}{3}(9-x^2)+x^2 e^{-\frac{x^2}{6}}\cdot\left(-\frac{1}{6}\right)2x\cdot\frac{1}{3}(9-x^2)+x^2 e^{-\frac{x^2}{6}}\cdot\frac{1}{3}(-2x) =$$

$$=\frac{2}{3}x e^{-\frac{x^2}{6}}(9-x^2)-\frac{1}{9}x^3 e^{-\frac{x^2}{6}}(9-x^2)-\frac{2}{3}x^3 e^{-\frac{x^2}{6}} = x e^{-\frac{x^2}{6}}\left(\frac{2}{3}(9-x^2)-\frac{1}{9}x^2(9-x^2)-\right.$$

$$\left.\frac{2}{3}x^2\right) = x e^{-\frac{x^2}{6}}\cdot\frac{54-6x^2-9x^2+x^4-6x^2}{9} = x e^{-\frac{x^2}{6}}\cdot\frac{x^4-21x^2+54}{9}$$

$y''=0$ akko $x=0$; $x^4-21x^2+54=0$ $\qquad t_{1,2}=\frac{21\pm 15}{2}$ $\qquad 3\sqrt{2}\approx 4{,}24$

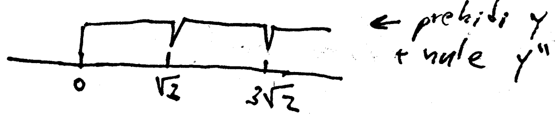
$x^2=t$

$t^2-21t+54=0$ $\qquad t_1=\frac{36}{2}=18 \quad t_2=\frac{6}{2}=3$

$D=441-216=225$ $\qquad x^2=18 \qquad x^2=3$

$x=\pm\sqrt{18} \qquad x_3=-\sqrt{3}$

f-ja simetrična u odnosu na koordinatni početak $\quad x_1=3\sqrt{2} \quad x_2=-3\sqrt{2} \quad x_4=\sqrt{3}\approx 1{,}73$ pa nas zanimaju samo pozitivne vrijednosti



x	$(0,\sqrt{3})$	$(\sqrt{3},3\sqrt{2})$	$(3\sqrt{2},+\infty)$
y''	+	−	+
y	∪	∩	∪
P.T.	P.T.	P.T.	

$y=x^3 e^{-\frac{x^2}{6}}$

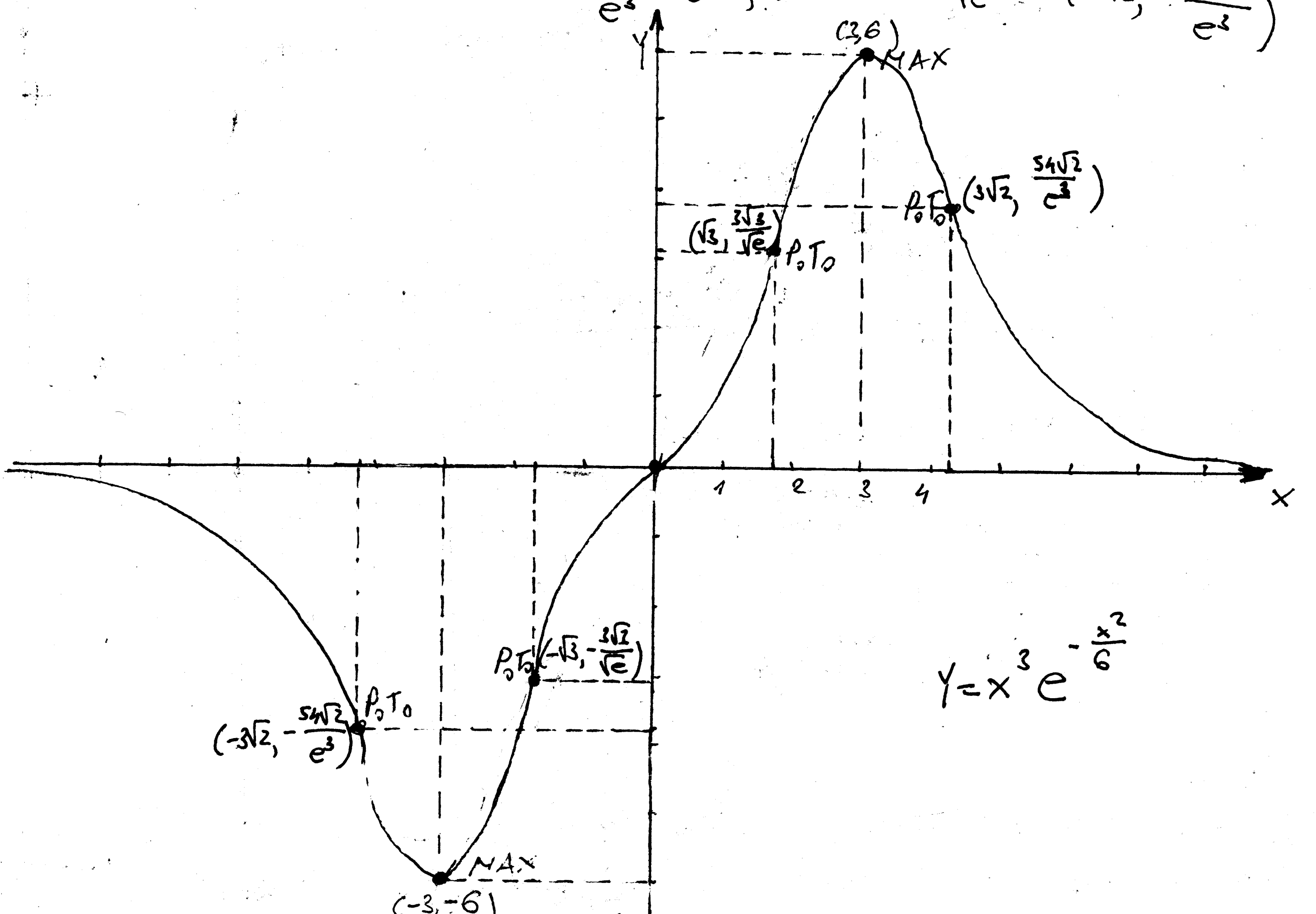
$y(0)=0$

$y(\sqrt{3})=3\sqrt{3}\,e^{-\frac{3}{6}}=\frac{3\sqrt{3}}{\sqrt{e}}\approx 3{,}15$

$y(3\sqrt{2})=27\cdot 2\sqrt{2}\,e^{-\frac{9\cdot 2}{6}}=54\sqrt{2}\,e^{-3}=\frac{54\sqrt{2}}{e^3}\approx 3{,}8$

Prevojne tačke su $(0,0)$, $\left(\sqrt{3},\frac{3\sqrt{3}}{\sqrt{e}}\right)$, $\left(3\sqrt{2},\frac{54\sqrt{2}}{e^3}\right)$, $\left(-\sqrt{3},-\frac{3\sqrt{3}}{\sqrt{e}}\right)$ i $\left(-3\sqrt{2},-\frac{54\sqrt{2}}{e^3}\right)$

grafik



(#) Ispitati i grafički predstaviti f-ju $y = \frac{1}{x}\ln x$.

Rj. definiciono područje

$x \neq 0, \quad x > 0$

$\mathcal{D}: x \in (0, +\infty)$

parnost neparnost periodičnost

$\mathcal{D}$ nije simetrično $\Rightarrow$

f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa y-osom, znak f-je

$y = 0$

$\frac{1}{x}\ln x = 0$

$\ln x = 0$

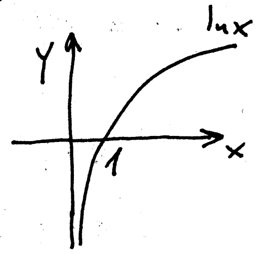
$x = e^0$

$x = 1$

(1,0) je nula f-je

f(0) nije definisano

f-ja ne siječe y-osu



x	(0, 1)	(1, +∞)
ln x	−	+
y	−	+

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

$$\lim_{x\to+0} f(x) = \lim_{x\to+0}\frac{1}{x}\ln x \;\left(=\infty\cdot(-\infty)\right) = \frac{1}{+0}\ln(+0) = (+\infty)\cdot(-\infty) = -\infty$$

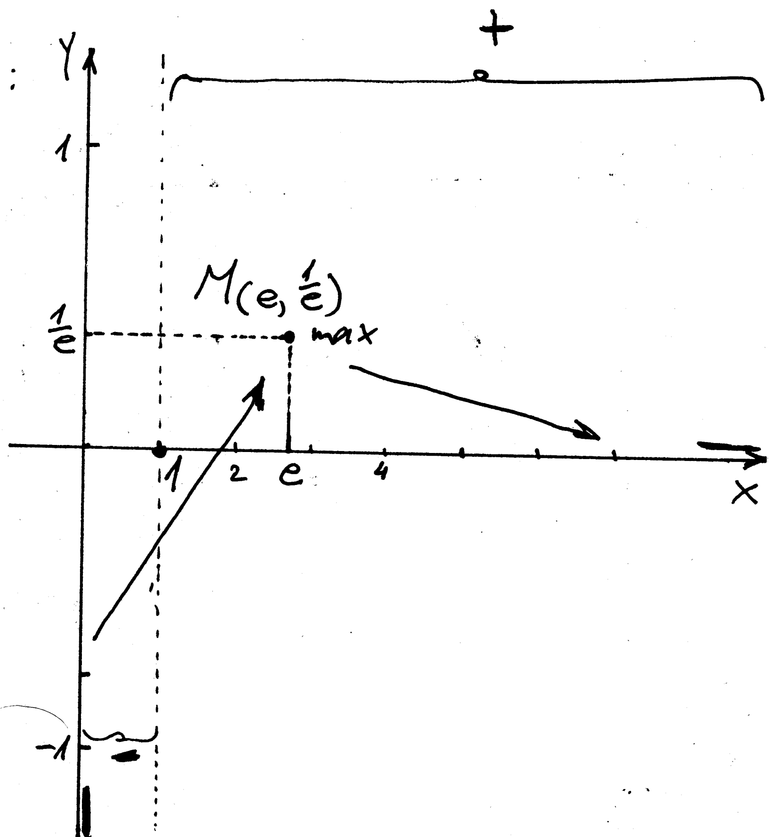
$\Rightarrow$ $x = 0$ je V. A. (sa desne strane)

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\frac{\ln x}{x}\;\left(=\frac{\infty}{\infty}\right) \overset{L.P.}{=} \lim_{x\to\infty}\frac{\frac{1}{x}}{1} = 0 \Rightarrow$$

$\Rightarrow$ $y = 0$ je H. A.

f-ja nema kosu asimptotu

počinjemo sa skiciranjem grafa:



rast i opadanje

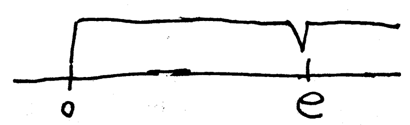
$$y' = \left(\frac{1}{x}\ln x\right)' = \left(\frac{\ln x}{x}\right)' =$$

$$= \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1-\ln x}{x^2}$$

$y' = 0$ akko $1 - \ln x = 0$

$\ln x = 1$

$x = e \approx 2{,}7183$



← nule y'
+ prekidi y

x	$(0, e)$	$(e, +\infty)$
y'	$+$	$-$
y	$\nearrow$	$\searrow$

max

rast i opadanje

$$f(e) = \frac{1}{e} \ln e = \frac{1}{e} \approx 0,3679$$

ekstremi f-je

Na osnovu tabele rasta i opadanja, f-ja ima maksimum u tački $M(e, \frac{1}{e})$.

prevojne tačke i intervali konveksnosti i konkavnosti.

$$y'' = \left(\frac{1-\ln x}{x^2} \right)' = \frac{-\frac{1}{x} \cdot x^2 - (1-\ln x)2x}{x^4} = \frac{-\cancel{x} - (1-\ln x) \cdot 2\cancel{x}}{x^{\cancel{4}\,3}} = \frac{-1-2+2\ln x}{x^3}$$

$$y'' = \frac{2\ln x - 3}{x^3},$$

$y'' = 0$ akko $2\ln x - 3 = 0$

$$2\ln x = 3$$

$$\ln x = \frac{3}{2}$$

$$x = e^{\frac{3}{2}} = \sqrt{e^3} \approx 4,4817$$

nule y'' + prekidi y

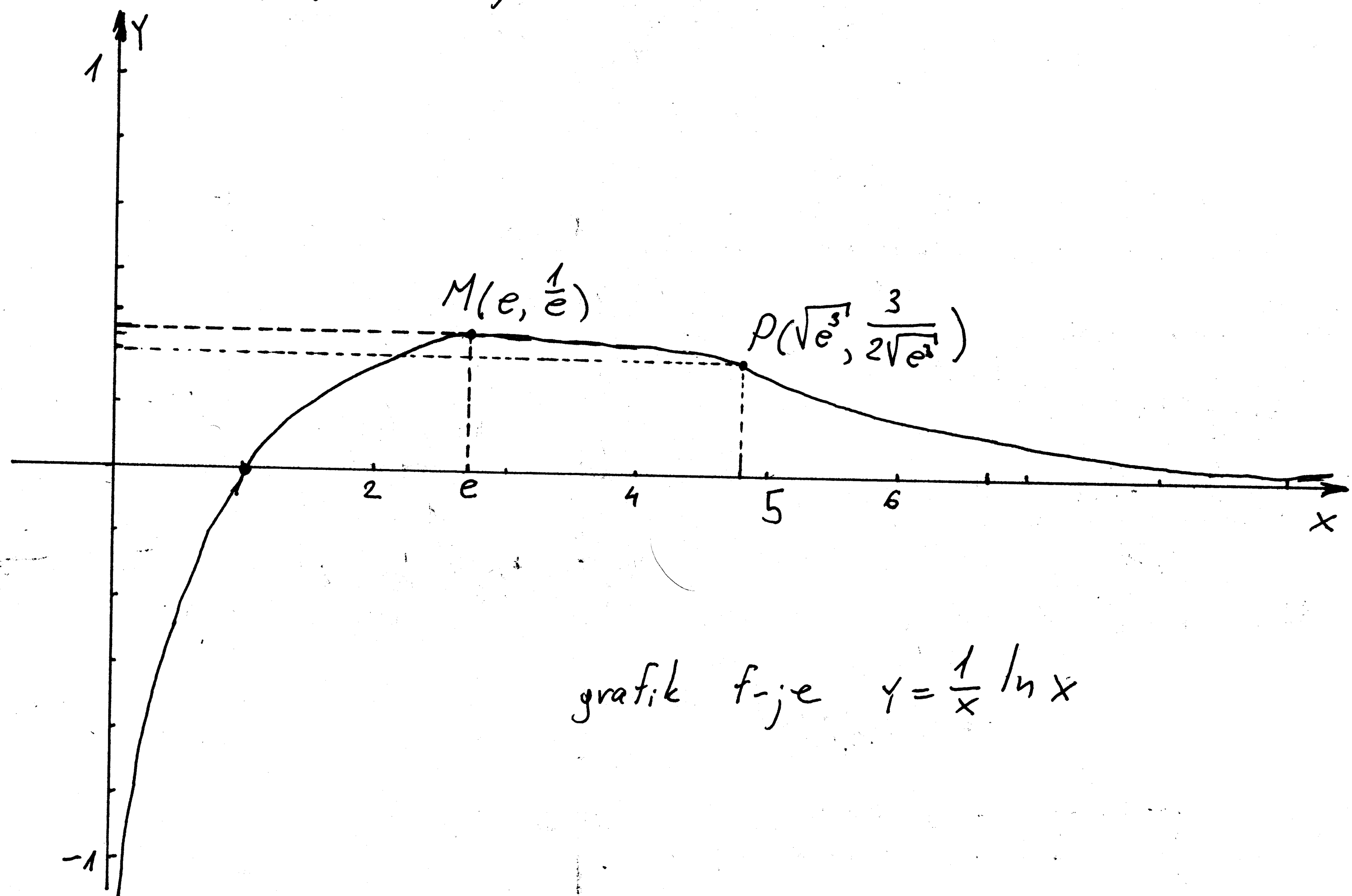
0 $\quad \sqrt{e^3}$

x	$(0, \sqrt{e^3})$	$(\sqrt{e^3}, +\infty)$
y''	$-$	$+$
y	$\frown$	$\cup$

P_0T_0

$$f(e^{\frac{3}{2}}) = \frac{1}{\sqrt{e^3}} \cdot \frac{3}{2} = \frac{3}{2\sqrt{e^3}} \approx 0,3347.$$

$P(\sqrt{e^3}, \frac{3}{2\sqrt{e^3}})$ je prevojna tačka



grafik f-je $y = \frac{1}{x} \ln x$

(#) Ispitati f-ju i nacrtati joj grafik $y = \dfrac{\ln x - 1}{x^3}$.

Rj: definiciono područje

$x \neq 0 \quad x > 0$

$\mathcal{D}: x \in (0, +\infty)$

parnost, neparnost, periodičnost

$\mathcal{D}$ nije simetrično $\Rightarrow$

$\Rightarrow$ f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa y-osom, znak f-je

$y = 0$ akko $\ln x - 1 = 0$

$\ln x = 1$

$x = e$

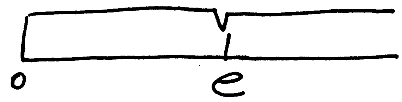
$(e, 0)$ nula f-je

$e \approx 2{,}7183$

$f(0) = ?$

$f(0)$ nije definisano

f-ja ne sijeće y-osu



x	$(0, e)$	$(e, +\infty)$
$\ln x - 1$	$-$	$+$
x^3	$+$	$+$
y	$-$	$+$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

$$\lim_{x\to+0} f(x) = \lim_{x\to+0} \frac{\ln x - 1}{x^3} \left(= \frac{-\infty - 1}{+0}\right) = \frac{-\infty}{+0} = -\infty \Rightarrow x = 0 \text{ je V. A.}$$

(sa desne strane)

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{\ln x - 1}{x^3} \left(= \frac{\infty}{\infty}\right) \overset{L.P.}{=} \lim_{x\to+\infty} \frac{\frac{1}{x}}{3x^2} = \lim_{x\to+\infty} \frac{1}{3x^3} = \frac{1}{\infty} = 0$$

$\Rightarrow y = 0$ je H. A.

f-ja nema K. A.

počinjemo sa skiciranjem grafa

rast i opadanje

$$y' = \left(\frac{\ln x - 1}{x^3}\right)' = \frac{\frac{1}{x} \cdot x^{\cancel{3}\,2} - (\ln x - 1) \cdot 3\cancel{x^2}}{x^{\cancel{6}\,4}}$$

$$y' = \frac{1 - 3\ln x + 3}{x^4} = \frac{4 - 3\ln x}{x^4}$$

$y' = 0$ akko $4 - 3\ln x = 0$

$3\ln x = 4$

$\ln x = \frac{4}{3}$ $\quad x = e^{\frac{4}{3}} = \sqrt[3]{e^4} \approx 3{,}7937$

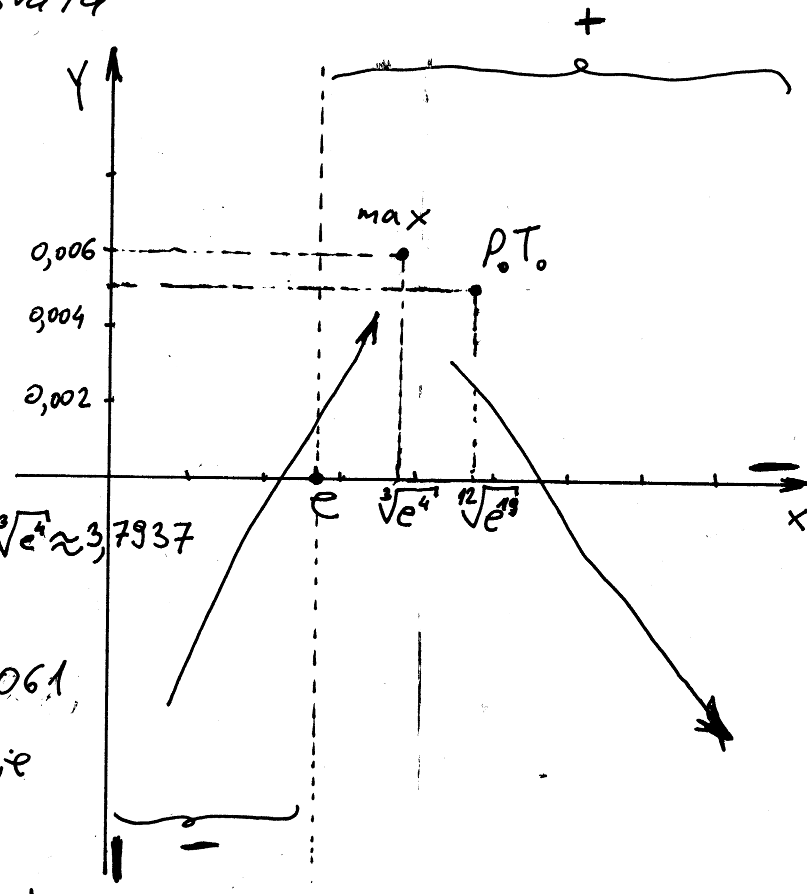
x	$(0, \sqrt[3]{e^4})$	$(\sqrt[3]{e^4}, +\infty)$
y'	$+$	$-$
y	$\nearrow$	$\searrow$

max

rast i opadanje

$\frac{1}{3e^4} \approx 0{,}0061$

$$f(e^{\frac{4}{3}}) = \frac{\ln e^{\frac{4}{3}} - 1}{(\sqrt[3]{e^4})^3} = \frac{\frac{4}{3} - 1}{(\sqrt[3]{e^4})^3} = \frac{\frac{1}{3}}{e^4} = \frac{1}{3e^4}$$



ekstremi f-je

na osnovu tabele rasta i opadanja tačka $M\left(\sqrt[3]{e^4}, \frac{1}{3e^4}\right)$ je tačka maksimuma.

prevojne tačke i intervali konveksnosti i konkavnosti

$$y'' = \left(\frac{4-3\ln x}{x^4}\right)' = \frac{-3\cdot\frac{1}{x}\cdot x^4 - (4-3\ln x)\cdot 4x^3}{(x^4)^2} = \frac{-3\cancel{x^3} - (4-3\ln x)\cdot 4\cancel{x^3}}{x^5\cdot\cancel{x^3}} = \frac{-3-16+12\ln x}{x^5}$$

$$y'' = \frac{12\ln x - 19}{x^5}$$

$y''=0$ akko $12\ln x - 19 = 0$

$12\ln x = 19$

$\ln x = \frac{19}{12}$

$x = e^{\frac{19}{12}} = \sqrt[12]{e^{19}} \approx 4,8712$

x	$(0, \sqrt[12]{e^{19}})$	$(\sqrt[12]{e^{19}}, +\infty)$
y''	−	+
y	⌒	∪

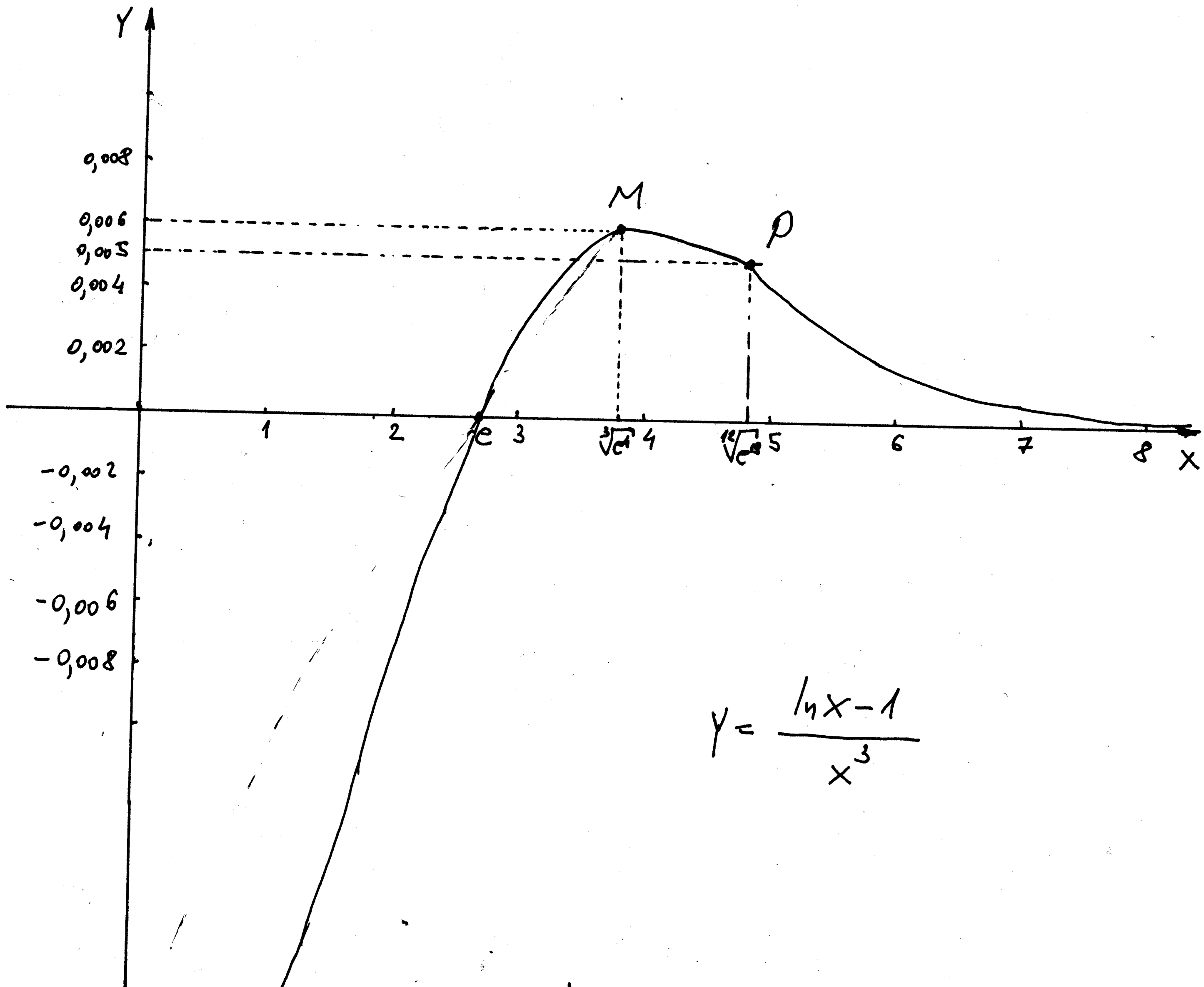
intervali konveksnosti i konkavnosti

P.T.

$$f(e^{\frac{19}{12}}) = \frac{\ln e^{\frac{19}{12}} - 1}{(e^{\frac{19}{12}})^3} = \frac{\frac{19}{12}-1}{e^{\frac{19}{4}}} = \frac{\frac{7}{12}}{e^{\frac{19}{4}}} = \frac{7}{12\sqrt[4]{e^{19}}} \approx 0,005$$

$P\left(\sqrt[12]{e^{19}}, \frac{7}{12\sqrt[4]{e^{19}}}\right)$ je prevojna tačka

grafik



#) Ispitati f-ju i nacrtati joj grafik $y = \frac{x}{x-1} \ln \frac{x}{x-1}$ (bez analize drugog izvoda).

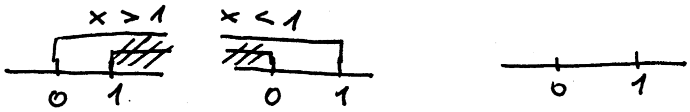
Rj: definiciono područje

$x-1 \neq 0$ $\quad \frac{x}{x-1} > 0$ $\quad \mathcal{D}: x \in (-\infty, 0) \cup (1, +\infty)$

$x \neq 1$

$x > 0$ $\quad x < 0$

$x > 1$ $\quad x < 1$



parnost, neparnost, periodičnost

$\mathcal{D}$ nije simetrično $\Rightarrow$

$\Rightarrow$ f-ja nije ni parna ni neparna

f-ja nije periodična

$\ln \frac{x}{x-1} > 0$ $\quad \frac{x}{x-1} - 1 > 0$

$\ln \frac{x}{x-1} > \ln 1$ $\quad \frac{x-x+1}{x-1} > 0$

$\frac{x}{x-1} > 1$ $\quad \frac{1}{x-1} > 0$

$x - 1 > 0$

$x > 1$

nule, presjek sa y-osom, znak f-je

$y = 0$ akko $x = 0$

za $x=0$ f-ja nije definisana

f-ja nema nulu i ne sijeće y-osu

x	$(-\infty, 0)$	$(1, +\infty)$
x	$-$	$+$
$x-1$	$-$	$+$
$\ln \frac{x}{x-1}$	$-$	$+$
y	$-$	$+$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

$$\lim_{x \to 0-} f(x) = \lim_{x \to 0-} \frac{x}{x-1} \ln \frac{x}{x-1} \left(= (0-)\cdot(-\infty)\right)$$

$$= \lim_{x \to 0-} \frac{\ln \frac{x}{x-1}}{\frac{x-1}{x}} \left(= \frac{-\infty}{\infty}\right) \stackrel{LoP}{=} \lim_{x \to 0-} \frac{\frac{1}{\frac{x}{x-1}}\left(\frac{x}{x-1}\right)'}{\left(\frac{x-x+1}{x^2}\right)} = \lim_{x \to 0-} \frac{\frac{\cancel{x-1}}{x} \cdot \frac{\cancel{x-1-x}}{(x-1)^{\cancel{2}}}}{\frac{1}{x^2}} = \lim_{x \to 0-} \frac{(-1)\, x^{\cancel{2}}}{\cancel{x}(x-1)} = 0$$

nema V. A. za $x=0$

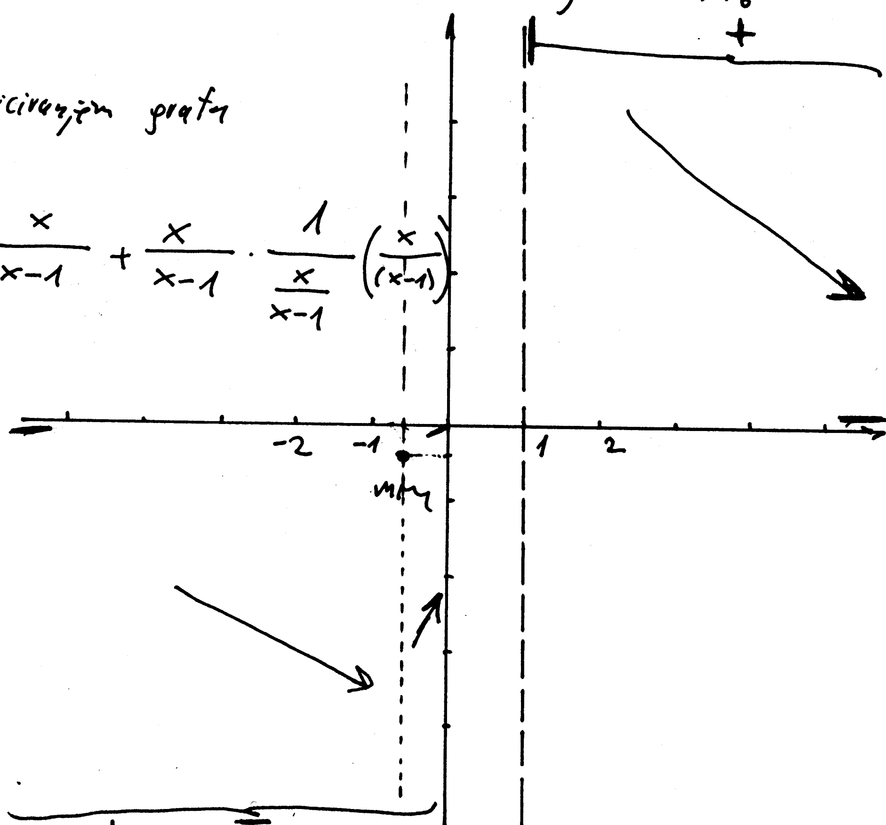
$$\lim_{x \to 1+0} f(x) = \lim_{x \to 1+0} \frac{x}{x-1} \ln \frac{x}{x-1} = (+\infty)\cdot(+\infty) = +\infty \Rightarrow x = 1 \text{ je V. A.}$$

$$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{x}{x-1} \ln \frac{x}{x-1} = \lim_{x \to +\infty} \frac{1}{1-\frac{1}{x}} \ln \frac{1}{1-\frac{1}{x}} = 1 \cdot \ln 1 = 1 \cdot 0 = 0 \Rightarrow y = 0 \text{ je H. A.}$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{1}{1-\frac{1}{x}} \ln \frac{1}{1-\frac{1}{x}} = 1 \cdot \ln 1 = 1 \cdot 0 = 0 \Rightarrow y = 0 \text{ je H. A.}$$

f-ja nema kose asimptote

nakon ovog koraka počinjemo sa skiciranjem grafa



rast i opadanje

$$y' = \left(\frac{x}{x-1} \ln \frac{x}{x-1}\right)' = \frac{x-1-x}{(x-1)^2} \ln \frac{x}{x-1} + \frac{x}{x-1} \cdot \frac{1}{\frac{x}{x-1}} \left(\frac{x}{x-1}\right)'$$

$$y' = \frac{-1}{(x-1)^2} \ln \frac{x}{x-1} + \frac{-1}{(x-1)^2}$$

$$y' = \frac{-1}{(x-1)^2} \left(\ln \frac{x}{x-1} + 1\right)$$

$y' = 0$ akko $\ln \frac{x}{x-1} + 1 = 0$

$\ln \frac{x}{x-1} = -1$

$\frac{x}{x-1} = e^{-1}$

$$\frac{x}{x-1} - \frac{1}{e} = 0$$

$$\frac{ex-(x-1)}{e(x-1)} = 0$$

$$ex - x + 1 = 0$$

$$x(e-1) = -1$$

$$x = -\frac{1}{e-1} \approx -0{,}5820$$

x	$(-\infty, -\frac{1}{e-1})$	$(-\frac{1}{e-1}, 0)$	$(1, +\infty)$
y'	$-$	$+$	$-$
y	$\searrow$	$\nearrow$	$\searrow$

rast i opadanje

$-\frac{1}{e-1}$ 0 1 ← prekidi y + nule y'

$$\ln \frac{-\frac{1}{e^{-1}-1}}{-\frac{1}{e^{-1}-1}-1} = e \qquad \ln\frac{5}{4} \approx 0{,}22$$

$$e > e^{-1}$$

$$e-1 > e^{-1}-1$$

$$\frac{1}{e-1} < \frac{1}{e^{-1}-1} \quad /\cdot(-1)$$

$$-\frac{1}{e-1} > -\frac{1}{e^{-1}-1}$$

$$\ln \frac{-\frac{1}{e-1}}{-\frac{1}{e-1}-1} = e^{-1}$$

$$f\left(-\frac{1}{e-1}\right) = \frac{-\frac{1}{e-1}}{-\frac{1}{e-1}-1} \ln \frac{\frac{-1}{e-1}}{\frac{-1-(e-1)}{e-1}} = \frac{\frac{-1}{e-1}}{\frac{-e}{e-1}} \ln\frac{1}{e} = \frac{1}{e}\cdot(-1) = -\frac{1}{e} \approx -0{,}3679$$

ekstremi f-je

Na osnovu tabele rasta i opadanja tačka minimuma je $(-\frac{1}{e-1}, -\frac{1}{e})$

prevojne tačke i intervali konveksnosti i konkavnosti

$$y'' = \left[-(x-1)^{-2}\left(\ln\frac{x}{x-1}+1\right)\right]' = 2(x-1)^{-3}\left(\ln\frac{x}{x-1}+1\right) + \left(-(x-1)^{-2}\right)\frac{x-1}{x}\cdot\frac{-1}{(x-1)^2}$$

$$y'' = 2(x-1)^{-3}\left(\ln\frac{x}{x-1}+1\right) - (x-1)^{-1}\cdot\frac{-1}{x(x-1)^2} = \frac{1}{(x-1)^3}\left[2\left(\ln\frac{x}{x-1}+1\right) + \frac{1}{x}\right]$$

bez analize drugog izvoda

(crtamo graf)

$$2\left(\ln\frac{x}{x-1}+1\right) + \frac{1}{x} = g(x)$$

$g(-2) \approx 0{,}6891$

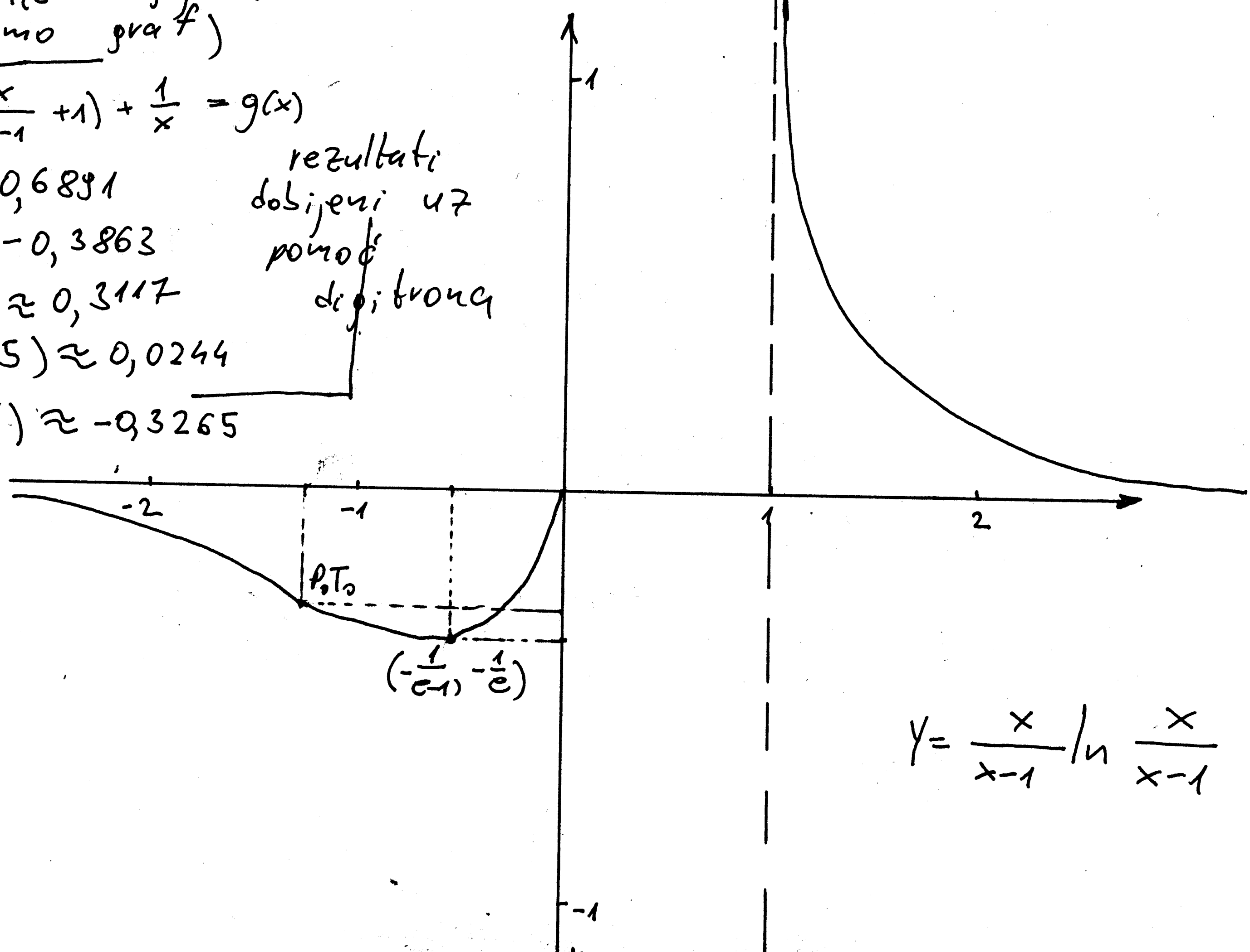
$g(-1) \approx -0{,}3863$

$g(-1{,}5) \approx 0{,}3117$

$g(-1{,}25) \approx 0{,}0244$

rezultati dobijeni uz pomoć digitrona

$f(-1{,}25) \approx -0{,}3265$



$$y = \frac{x}{x-1}\ln\frac{x}{x-1}$$

(#) Ispitati f-ju i nacrtati njen grafik $\quad Y = \dfrac{x^2+10}{x^2+4x+4}$.

Rj. $Y = \dfrac{x^2+10}{x^2+4x+4} = \dfrac{x^2+10}{(x+2)^2}$

definiciono područje

$x+2 \neq 0 \qquad \mathcal{D}: x \in (-\infty, -2) \cup (-2, +\infty)$

$x \neq -2$

nule, presjek sa Y-osom i znak f-je

$Y=0 \Rightarrow x^2+10=0$

Kako je $x^2+10>0 \ \forall x \in \mathcal{D}$ to f-ja nema nule

$f(0) = \dfrac{0+10}{(0+2)^2} = \dfrac{10}{4} = \dfrac{5}{2}$

$(0, \frac{5}{2})$ je presjek sa Y-osom

$x^2+10>0 \ \forall x \in \mathcal{D}$
$(x+2)^2>0 \ \forall x \in \mathcal{D}$ f-ja je uvijek pozitivna

·parnost (neparnost), periodičnost

$\mathcal{D}$ nije simetrično $\Rightarrow$ f-ja nije ni parna ni neparna

f-ja nije periodična

ponašanje na krajevima intervala definisanosti i asimptote

za $x=-2$ f-ja ima prekid

$\lim\limits_{x\to -2-0} f(x) = \lim\limits_{x\to -2-0} \dfrac{x^2+10}{(x+2)^2} = \dfrac{(-2-0)^2+10}{(-2-0+2)^2} = \dfrac{14+0}{+0} = +\infty \Rightarrow x=-2$ je V. A. (sa lijeve strane)

$\lim\limits_{x\to -2+0} f(x) = \lim\limits_{x\to -2+0} \dfrac{x^2+10}{(x+2)^2} = \dfrac{(-2+0)^2+10}{(-2+0+2)^2} = \dfrac{14-0}{+0} = +\infty \Rightarrow x=-2$ je V. A. (sa desne strane)

$\lim\limits_{x\to \mp\infty} f(x) = \lim\limits_{x\to \mp\infty} \dfrac{x^2+10 \ :x^2}{x^2+4x+4 :x^2} = \lim\limits_{x\to \mp\infty} \dfrac{1+\overset{\to 0}{\frac{10}{x^2}}}{1+\underset{\to 0}{\frac{4}{x}}+\underset{\to 0}{\frac{4}{x^2}}} = 1 \Rightarrow Y=1$ je H. A.

f-ja nema kasu asimptotu

Poslije ovog koraka počinjemo skicirati grafik.

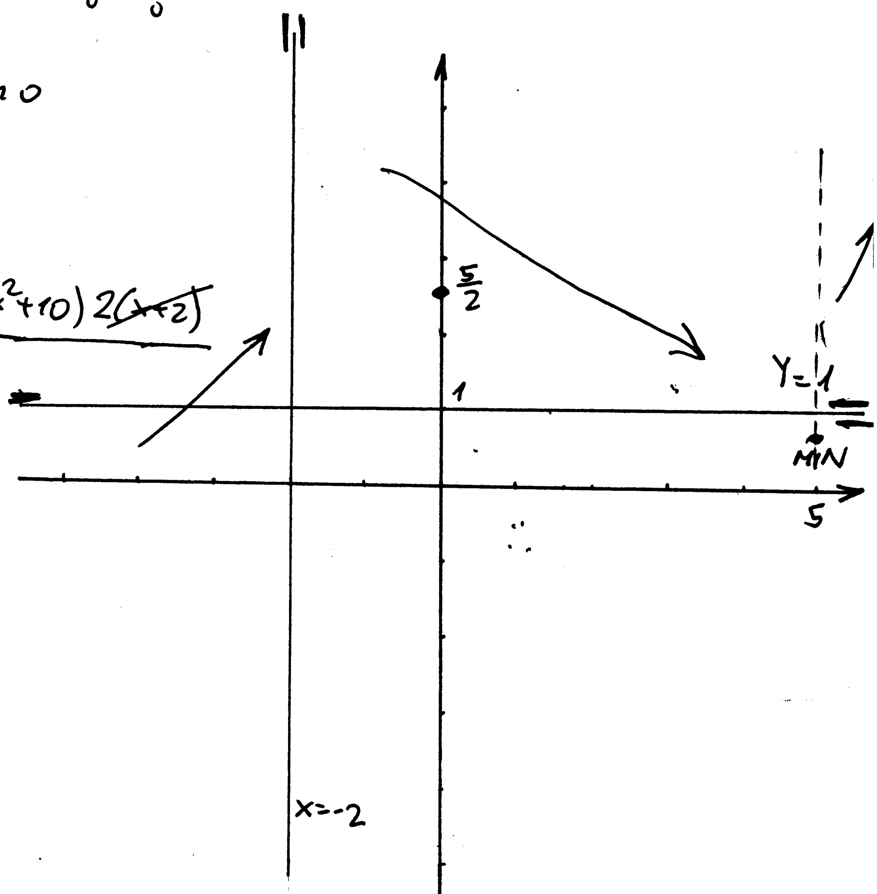
rast i opadanje

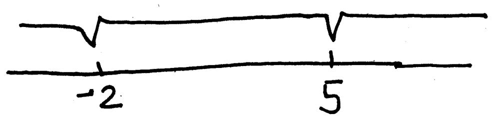
$Y' = \left(\dfrac{x^2+10}{(x+2)^2}\right)' = \dfrac{2x\cdot(x+2)^{\cancel{2}} - (x^2+10)\,2\cancel{(x+2)}}{(x+2)^{\cancel{4}\,3}}$

$Y' = \dfrac{\cancel{2x^2}+4x-\cancel{2x^2}-20}{(x+2)^3}$

$Y' = \dfrac{4x-20}{(x+2)^3} = 4\,\dfrac{x-5}{(x+2)^3}$

$Y'=0$ akko $x-5=0$
$x=5$





← prekidi y + nule y'

x	$(-\infty,-2)$	$(-2,5)$	$(5,+\infty)$
y'	+	—	+
y	↗	↘	↗
		min	

rast i opadanje

ekstremi f-je

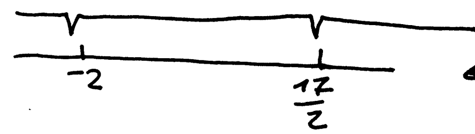
Stacionarna tačka je $x=5$.
Na osnovu tabele rasta i opadanja vidimo da f-ja u toj tački ima ekstrem i to minimum

$$f(5)=\frac{25+10}{7^2}=\frac{35}{49}\approx 0,71 \qquad \left(5,\frac{35}{49}\right) \text{ je tačka minimuma}$$

prevojne tačke i intervali konveksnosti i konkavnosti

$$y''=\left(4\,\frac{x-5}{(x+2)^3}\right)'=4\,\frac{1\cdot(x+2)^{\cancel{3}}-(x-5)3\cancel{(x+2)^2}}{(x+2)^{\cancel{6}4}}=4\,\frac{x+2-3x+15}{(x+2)^4}.$$

$$y''=4\,\frac{-2x+17}{(x+2)^4}=-4\,\frac{2x-17}{(x+2)^4}$$



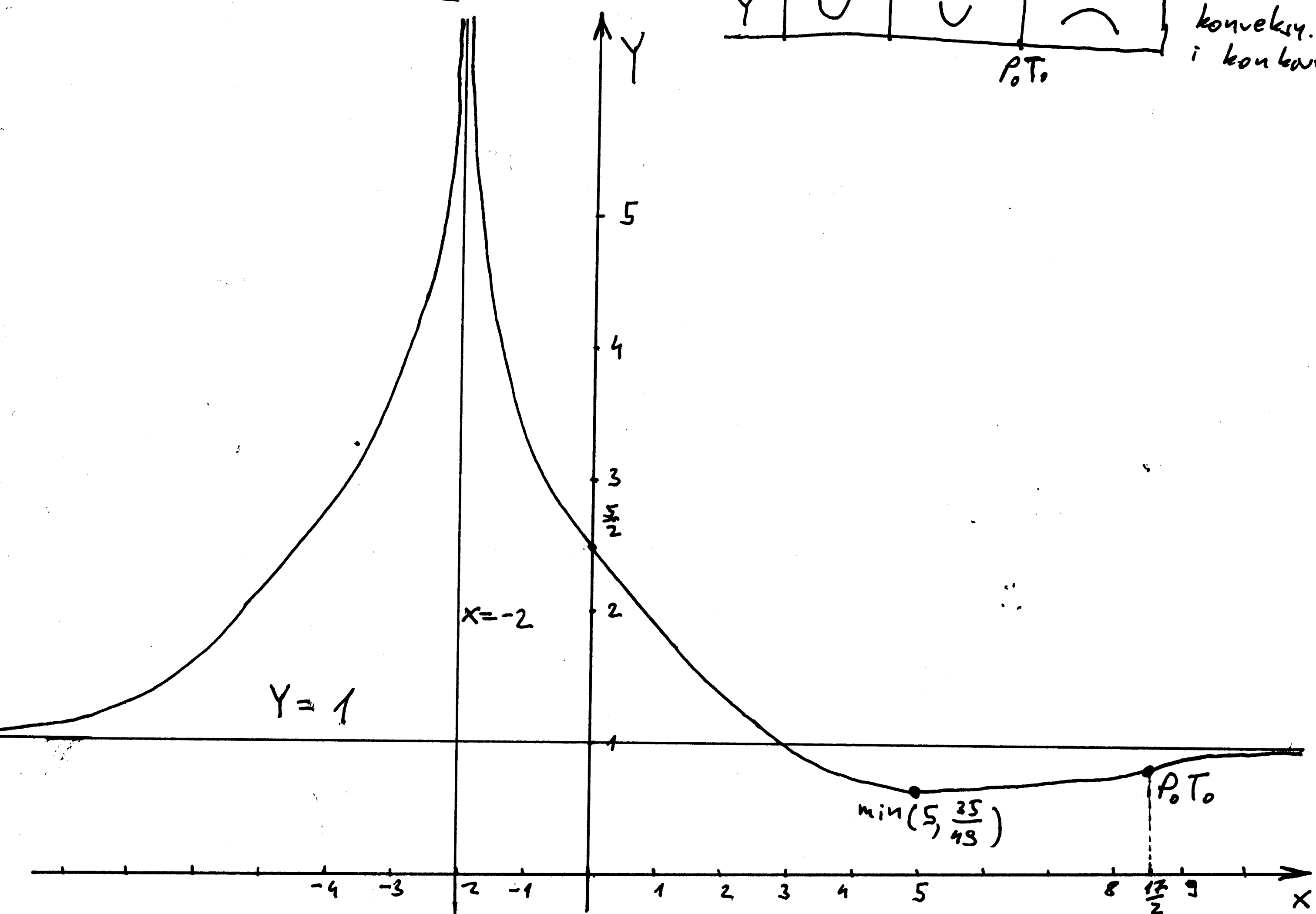
← nule y'' + prekidi y

$y''=0$ akko $2x-17=0$

$x=\frac{17}{2}$

x	$(-\infty,-2)$	$\left(-2,\frac{17}{2}\right)$	$\left(\frac{17}{2},+\infty\right)$
y''	+	+	—
y	∪	∪	⌒
			P_oT_o

intervali konveksn. i konkavn.



(#) Ispitati f-ju i nacrtati njen grafik: $y = \frac{x^3-2}{2x^2}$.

Rj. definiciono područje

$D: x \neq 0$

parnost (neparnost), periodičnost

$f(-x) = \frac{(-x)^3-2}{2(-x)^2} = \frac{-x^3-2}{2x^2} \neq \pm f(x)$

- f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa y-osom, ~~znak~~

$y=0$ akko $x^3-2=0$

$x = \sqrt[3]{2} \approx 1{,}26$

$(\sqrt[3]{2}, 0)$ je nula f-je

$f(0) =$ nije definisano

f-ja ne siječe y-osu

$2x^2 > 0 \quad \forall x \in D$

$$\left.\begin{array}{lll} y>0 & \text{za} & x > \sqrt[3]{2} \\ y<0 & \text{za} & x < \sqrt[3]{2} \end{array}\right\} \text{znak f-je.}$$

ponašanje na krajevima intervala definisanosti i asimptote

- za $x=0$ f-ja ima prekid

$$\left.\begin{array}{l} \lim\limits_{x\to 0-} f(x) = \lim\limits_{x\to 0-} \frac{x^3-2}{2x^2} = \frac{(0-)^3-2}{2(0-)^2} = \frac{-2-0}{0+} = -\infty \\ \lim\limits_{x\to 0+} f(x) = \lim\limits_{x\to 0+} \frac{(0+)^3-2}{2(0+)^2} = \frac{-2+0}{+0} = -\infty \end{array}\right\} \Rightarrow x=0 \text{ je V. A.}$$

$\lim\limits_{x\to\pm\infty} f(x) = \lim\limits_{x\to\pm\infty} \frac{x^3-2 \ /:x^2}{2x^2 \ /:x^2} = \pm\infty$ f-ja nema H. A.

Tražimo kosu asimptotu u obliku $y = kx+n$.

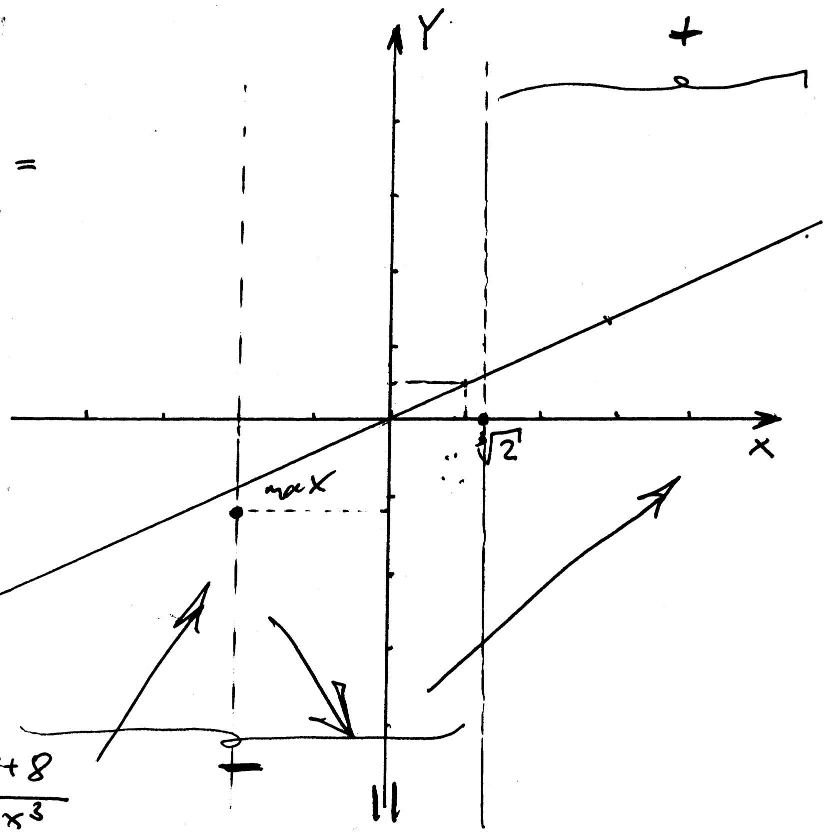
$k = \lim\limits_{x\to\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty} \frac{x^3-2 \ /:x^3}{2x^3 \ /:x^3} = \frac{1}{2}$

$n = \lim\limits_{x\to\infty} [f(x) - kx] = \lim\limits_{x\to\infty} \left[\frac{x^3-2}{2x^2} - \frac{1}{2}x\right] =$

$= \lim\limits_{x\to\infty} \frac{x^3-2-x^3}{2x^2} = \lim\limits_{x\to\infty} \frac{-2}{2x^2} = 0$

kosa asimptota je $y = \frac{1}{2}x$

Poslije ovog koraka počinjemo skicirati grafik.



rast i opadanje

$y' = \left(\frac{x^3-2}{2x^2}\right)' = \frac{3x^2 \cdot 2x^2 - (x^3-2)4x}{4x^4} =$

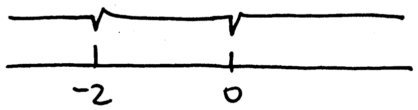
$= \frac{6x^4-4x^4+8x}{4x^4} = \frac{2x^4+8x}{4x^4} = \frac{x^3+8}{2x^3}$

$y' = \dfrac{x^3+8}{2x^3}$, $\quad y'=0$ akko $x^3+8=0$

$x^3 = -8$

$x = -2$

prekidi y + nule y'



x	$(-\infty,-2)$	$(-2,0)$	$(0,+\infty)$
y'	+	−	+
y	↗	↘	↗
		max	N.D.

$f(-2) = \dfrac{(-2)^3-2}{2(-2)^2} = \dfrac{-10}{8} = -\dfrac{5}{4} \approx -1{,}25$

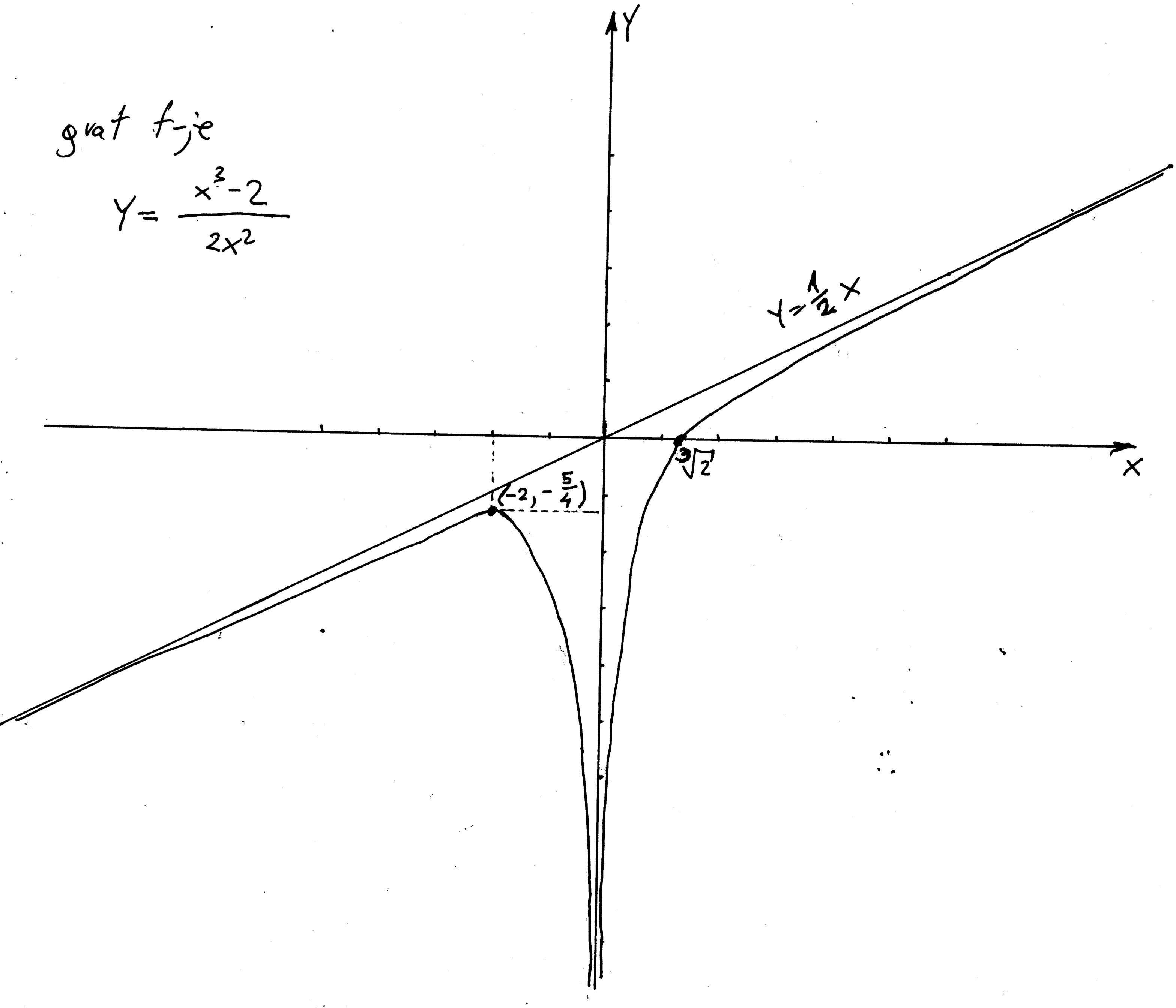
prevojne tačke i intervali konveksnosti i konkavnosti

$y'' = \left(\dfrac{x^3+8}{2x^3}\right)' = \dfrac{3x^2\cdot 2x^3-(x^3+8)\cdot 6x^2}{4x^6} = \dfrac{6x^5-6x^5-48}{4x^6} = \dfrac{-48}{4x^6} = -\dfrac{12}{x^6} < 0$

F-ja nema prevojnih tački i uvijek je negativna što znači uvijek je $\cap$ oblika.

graf f-je

$y = \dfrac{x^3-2}{2x^2}$



(#) Ispitati f-ju i nacrtati njen grafik $y = e^{\frac{x}{1-x}} - 1$.

Rj.

definiciono područje

$1 - x \neq 0$

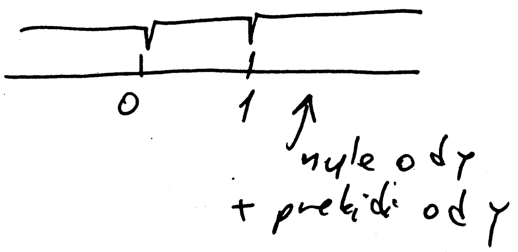
$x \neq 1$ $\qquad \mathcal{D}: x \in (-\infty, 1) \cup (1, +\infty)$

parnost (neparnost), periodičnost

$\mathcal{D}$ nije simetrično $\Rightarrow$

f-ja nije ni parna ni neparna

f-ja nije periodična



nule, presjek sa y-osom, znak f-je

$y = 0$ ako $e^{\frac{x}{1-x}} = 1$

tj. $\frac{x}{1-x} = 0 \Rightarrow x = 0$

$(0,0)$ je nula f-je i presjek sa y-osom

$y > 0 \Leftrightarrow e^{\frac{x}{1-x}} - 1 > 0$

	$(-\infty, 0)$	$(0, 1)$	$(1, +\infty)$
x	$-$	$+$	$+$
$1-x$	$+$	$+$	$-$
y	$-$	$+$	$-$

znak f-je

$e^{\frac{x}{1-x}} > 1$

$e^{\frac{x}{1-x}} > e^0$

$\frac{x}{1-x} > 0$

ponašanje na krajevima intervala definisanosti i asimptote

za $x = 1$ f-ja ima prekid

$$\lim_{x \to 1-0} f(x) = \lim_{x \to 1-0} \left(e^{\frac{x}{1-x}} - 1\right) = e^{\frac{1-0}{1-1+0}} - 1 = e^{\frac{1-0}{+0}} - 1 = e^{\infty} - 1 = \infty$$

$$\lim_{x \to 1+0} f(x) = \lim_{x \to 1+0} \left(e^{\frac{x}{1-x}} - 1\right) = e^{\frac{1+0}{1-1-0}} - 1 = e^{\frac{1+0}{-0}} - 1 = e^{-\infty} - 1 = \frac{1}{e^{\infty}} - 1 = -1$$

$x = 1$ je vertikalna asimptota (sa lijeve strane)

$$\lim_{x \to \mp\infty} f(x) = \lim_{x \to \mp\infty} \left(e^{\frac{x}{1-x} \,:x \atop :x} - 1\right) =$$

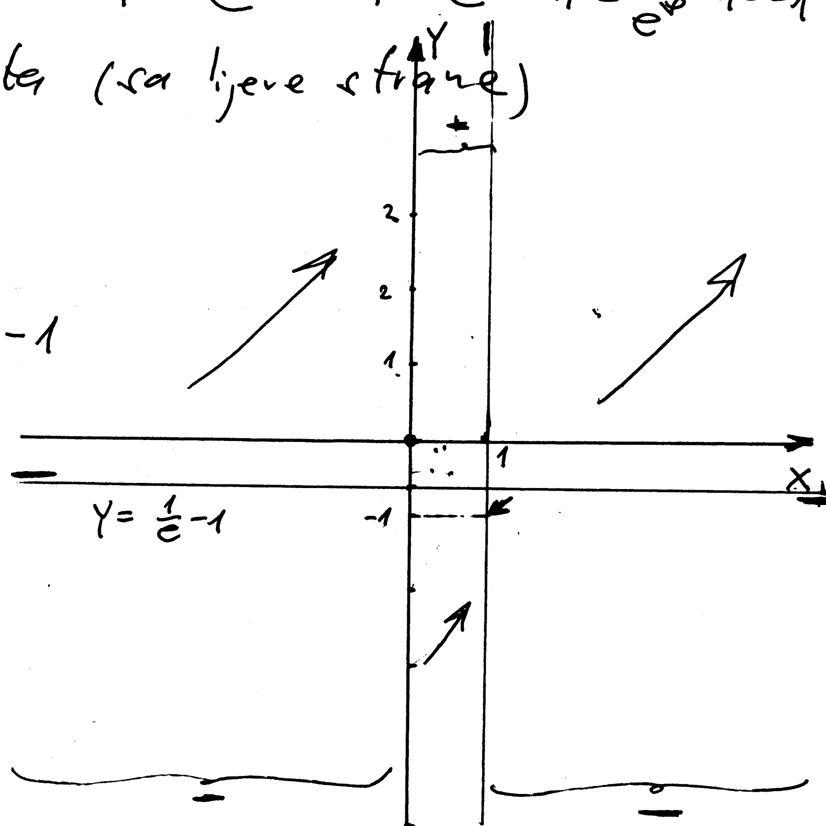
$$= \lim_{x \to \mp\infty} \left(e^{\frac{1}{\frac{1}{x} - 1}} - 1\right) = e^{-1} - 1 = \frac{1}{e} - 1$$

$y = \frac{1}{e} - 1 \approx -0{,}63$

je H. A.

kose asimptote nema

Poslije ovog koraka počinjemo sa skiciranjem grafa f-je



rast i opadanje

$$y' = \left(e^{\frac{x}{1-x}} - 1\right)' = e^{\frac{x}{1-x}} \cdot \left(\frac{x}{1-x}\right)' = \frac{1\cdot(1-x) - x\cdot(-1)}{(1-x)^2}\, e^{\frac{x}{1-x}} = \frac{e^{\frac{x}{1-x}}}{(1-x)^2}$$

$y' = \frac{1}{(1-x)^2} e^{\frac{x}{1-x}}$ $\quad y' > 0$ za $\forall x \in D$, f-ja $\nearrow$ za $\forall x$

ekstremi f-je

$y' \neq 0 \ \forall x$ f-ja nema ekstrema

$$y'' = \left(\frac{1}{(1-x)^2} e^{\frac{x}{1-x}}\right)' = (-2)(1-x)^{-3}(-1)e^{\frac{x}{1-x}} + \frac{1}{(1-x)^2} \cdot \frac{1}{(1-x)^2} e^{\frac{x}{1-x}}$$

$$y'' = \frac{2\cdot(1-x) + 1}{(1-x)^4} e^{\frac{x}{1-x}} = \frac{-2x+3}{(1-x)^4} e^{\frac{x}{1-x}}$$ $\quad y''=0$ akko $x = \frac{3}{2}$

prekidi od y + nule y''

$\longrightarrow$ $1 \quad \frac{3}{2}$

$f\left(\frac{3}{2}\right) = e^{\frac{\frac{3}{2}}{1-\frac{3}{2}}} - 1 = e^{\frac{\frac{3}{2}}{-\frac{1}{2}}} - 1$

$f\left(\frac{3}{2}\right) = e^{-3} - 1 \approx -0{,}95$

x	$(-\infty, 1)$	$(1, \frac{3}{2})$	$(\frac{3}{2}, +\infty)$
y''	$+$	$+$	$-$
y	$\cup$	$\cup$	$\cap$

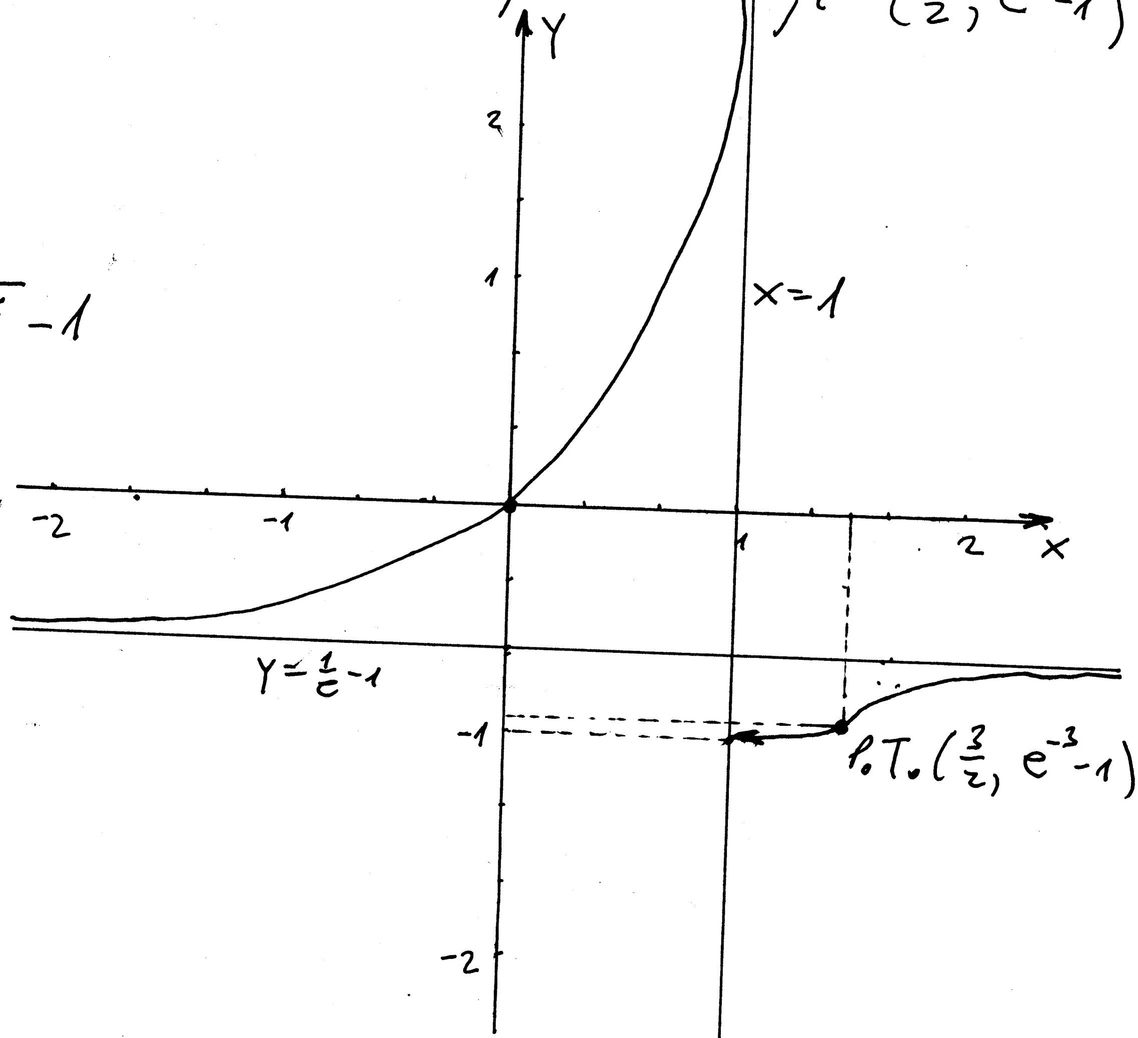
konveksnost i konkavnost

P.T.

Prevojna tačka je $\left(\frac{3}{2}, e^{-3}-1\right)$

grafik f-je

$y = e^{\frac{x}{1-x}} - 1$



Ispitati f-ju i nacrtati njen grafik: $y=\dfrac{\ln^2 x+1}{x^2}$.

Rj.: definiciono područje

$x\neq 0$ i $x>0$

$\mathcal{D}:\ x\in(0,+\infty)$

parnost (neparnost), periodičnost

. $\mathcal{D}$ nije simetrično

$\Rightarrow$ f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa Y-osom, znak f-je

$y=0$ akko $\ln^2 x+1=0$

$(\ln x)^2=-1$

f-ja nema nulu

$f(0)$ nije definisano

f-ja ne siječe Y-osu

$\ln^2 x+1>0\ \ \forall x\in\mathcal{D}$

$x^2>0\ \ \forall x\in\mathcal{D}$

f-ja je uvijek pozitivna

ponašanje na krajevima intervala definisanosti i asimptote

za $x\leq 0$ f-ja nije definisana

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}\frac{\ln^2 x+1}{x^2}=\frac{+\infty}{0^+}=+\infty \quad\Rightarrow\quad x=0 \text{ je vertikalna asimptota}$$

$$\lim_{x\to+\infty} f(x)=\lim_{x\to+\infty}\frac{\ln^2 x+1}{x^2}\left(=\frac{\infty}{\infty}\right)\overset{L.P.}{=}\lim_{x\to+\infty}\frac{2\ln x\cdot\frac{1}{x}}{2x}=\lim_{x\to+\infty}\frac{\ln x}{x^2}\left(=\frac{\infty}{\infty}\right)\overset{L.P.}{=}$$

$$=\lim_{x\to\infty}\frac{\frac{1}{x}}{2x}=0 \quad\Rightarrow\quad y=0 \text{ je horizontalna asimptota}$$

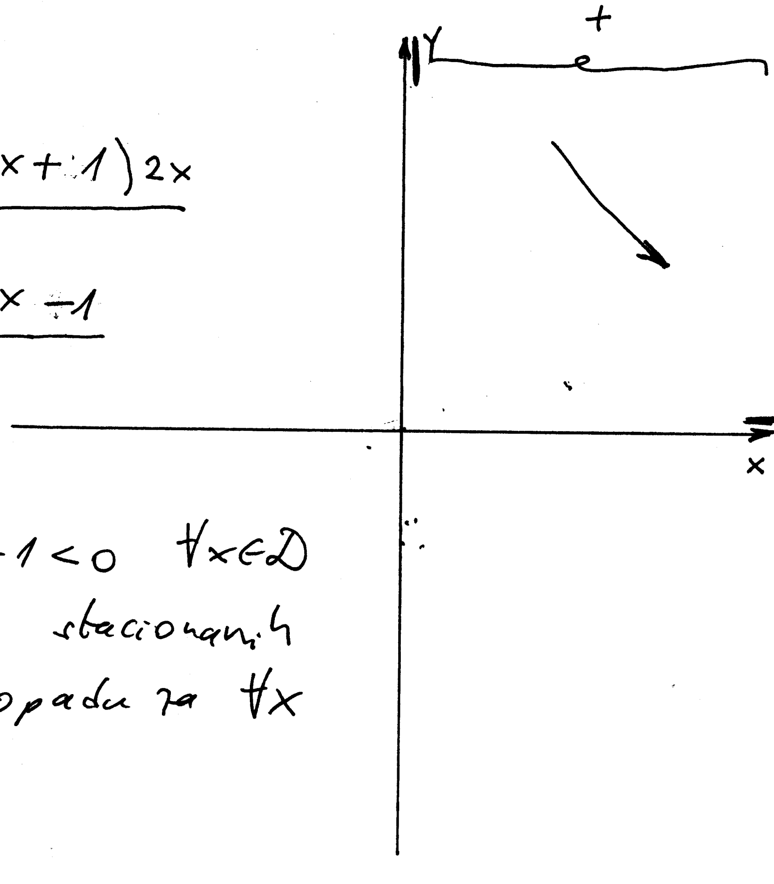
f-ja nema kosu asimptotu

počinjemo skicirati grafik

rast i opadanje

$$y'=\left(\frac{\ln^2 x+1}{x^2}\right)'=\frac{2\ln x\cdot\frac{1}{x}\cdot x^2-(\ln^2 x+1)2x}{x^4}$$

$$=\frac{2x(\ln x-\ln^2 x-1)}{x^4}=2\,\frac{\ln x-\ln^2 x-1}{x^3}$$



$y'=0$ akko $-\ln^2 x+\ln x-1=0$

$\ln x=t$

$-t^2+t-1=0$

$t^2-t+1=0$

$D=1-4<0$

$-\ln^2 x+\ln x-1<0\ \ \forall x\in\mathcal{D}$

f-ja nema stacionarnih tački i opada za $\forall x$

ekstremi f-je

f-ja nema stacionarnih tački $\Rightarrow$ f-ja nema ekstrema

prevojne tačke i intervali konveksnosti i konkavnosti

$$y'' = 2\left(\frac{\ln x - \ln^2 x - 1}{x^3}\right)' = 2\,\frac{\left(\frac{1}{x} - 2\ln x\cdot\frac{1}{x}\right)\cancel{x^3} - (\ln x - \ln^2 x - 1)\cdot 3\cancel{x^2}}{\cancel{x^6}\,x^4} =$$

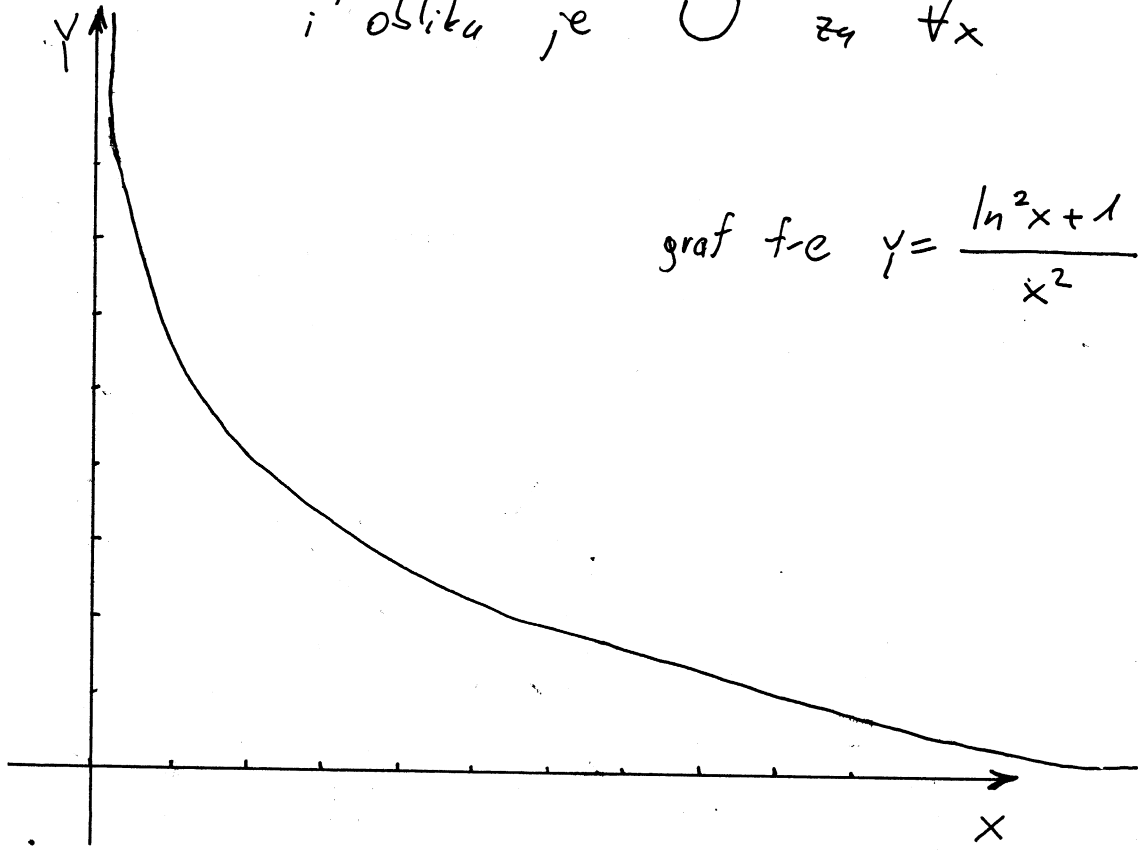
$$= 2\,\frac{1 - 2\ln x - 3\ln x + 3\ln^2 x + 3}{x^4} = 2\,\frac{3\ln^2 x - 5\ln x + 4}{x^4}$$

$$3\ln^2 x - 5\ln x + 4 = 0$$

$\ln x = t \qquad 3t^2 - 5t + 4 = 0 \qquad \Rightarrow \qquad 3\ln^2 x - 5\ln x + 4 > 0 \quad \forall x$

$D = 25 - 48 < 0 \qquad\qquad x^4 > 0 \quad \forall x$

$y'' > 0 \quad \forall x \in D \quad \Rightarrow$ f-ja nema prevojnih tački i oblika je $\cup$ za $\forall x$



graf f-je $y = \dfrac{\ln^2 x + 1}{x^2}$

(#) Ispitati f-ju i nacrtati joj grafik $\quad y = \dfrac{x^4-9x^2+12}{3x}$.

R: definiciono područje

$\mathcal{D}: x \neq 0$

$x \in \mathbb{R}\setminus\{0\}$

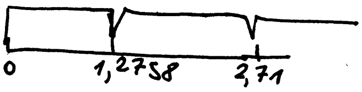
parnost (neparnost), periodičnost

$$f(-x) = \frac{(-x)^4 - 9(-x)^2+12}{3(-x)} = -\frac{x^4-9x^2+12}{3x} = -f(x)$$

f-ja je neparna $\xrightarrow{\text{simetrična u odnosu na } (0,0)}$ (dovoljno ju je ispitati za $x>0$)

f-ja nije periodična

znak f-je



← prekidi f-je i nule f-je

x	$(0, 1{,}27)$	$(1{,}27,\ 2{,}71)$	$(2{,}71, +\infty)$
y	+	−	+

znak f-je

nule, presjek sa y-osom i znak f-je

$y=0$ akko $x^4-9x^2+12=0$

$x^2=t \qquad t^2-9t+12=0$

$D = 81-48 = 33$

$$t_{1,2} = \frac{9 \pm \sqrt{33}}{2}$$

$x^2 = \dfrac{9-\sqrt{33}}{2} \qquad x^2 = \dfrac{9+\sqrt{33}}{2}$

$x_1 \approx -1{,}2758 \qquad x_2 \approx -2{,}7152$

$x_2 \approx 1{,}2758 \qquad x_4 \approx 2{,}7152$

$f(0)$ nije definisano

f-ja ne siječe y-osu

ponašanje na krajevima intervala definisanosti i asimptote

Za $x=0$ f-ja ima prekid

$$\left.\begin{aligned} \lim_{x\to 0+} f(x) &= \lim_{x\to +0} \frac{x^4-9x^2+12}{3x} = \frac{12}{0+} = +\infty \\ \lim_{x\to 0-} f(x) &= \lim_{x\to 0-} \frac{x^4-9x^2+12}{3x} = \frac{12}{0-} = -\infty \end{aligned}\right\} \Rightarrow \quad x=0 \text{ je V. A.}$$

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{x^4-9x^2+12}{3x} \,\Big/ {:x} = \lim_{x\to\pm\infty} \frac{x^3-9x+\frac{12}{x}}{3} = \pm\infty \Rightarrow \text{ f-ja nema H. A.}$$

tražimo kosu asimptotu u obliku $y=kx+n$,

$$k = \lim_{x\to\infty} \frac{f(x)}{x} = \lim_{x\to\infty} \frac{x^4-9x^2+12}{3x^2} \,\Big/ {:x^2} = \infty$$

f-ja nema kosu asimptotu

Nakon ovog koraka počinjemo skicirati graf f-je.

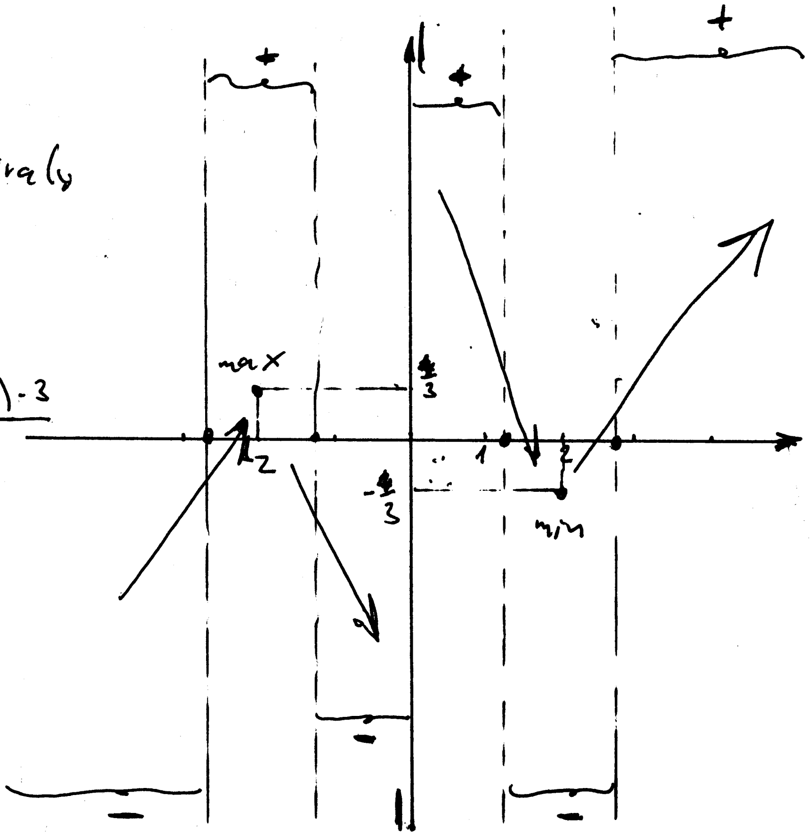
rast i opadanje

$$y' = \left(\frac{x^4-9x^2+12}{3x}\right)' = \frac{(4x^3-18x)3x-(x^4-9x^2+12)\cdot 3}{9x^2}$$

$$= \frac{12x^4-54x^2-3x^4+27x^2+36}{9x^2} =$$

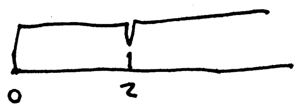
$$= \frac{9x^4-27x^2-36}{9x^2} = \frac{x^4-3x^2-4}{x^2}$$

$$y' = x^2-3-\frac{4}{x^2}$$



$y' = 0$ akko $x^4 - 3x^2 - 4 = 0$

$t = x^2$



← prekidi f-je y + nule f-je y'

$t^2 - 3t - 4 = 0$

$D = 9 + 16 = 25$

$t_{1,2} = \frac{3 \pm 5}{2}$

$t_1 = -1 \quad t_2 = 4$

$\Downarrow$

$x^2 = 4$

$x_1 = -2 \quad x_2 = 2$

x	(0,2)	(2,+∞)
y'	−	+
y	↘	↗

min

$f(2) = \frac{16 - 36 + 12}{6}$

$f(2) = -\frac{8}{6} = -\frac{4}{3}$

ekstremi f-je

Na osnovu tabele rasta i opadanja i simetričnosti grafa f-ja ima minimum u $(2, -\frac{4}{3})$ i maksimum u $(-2, \frac{4}{3})$.

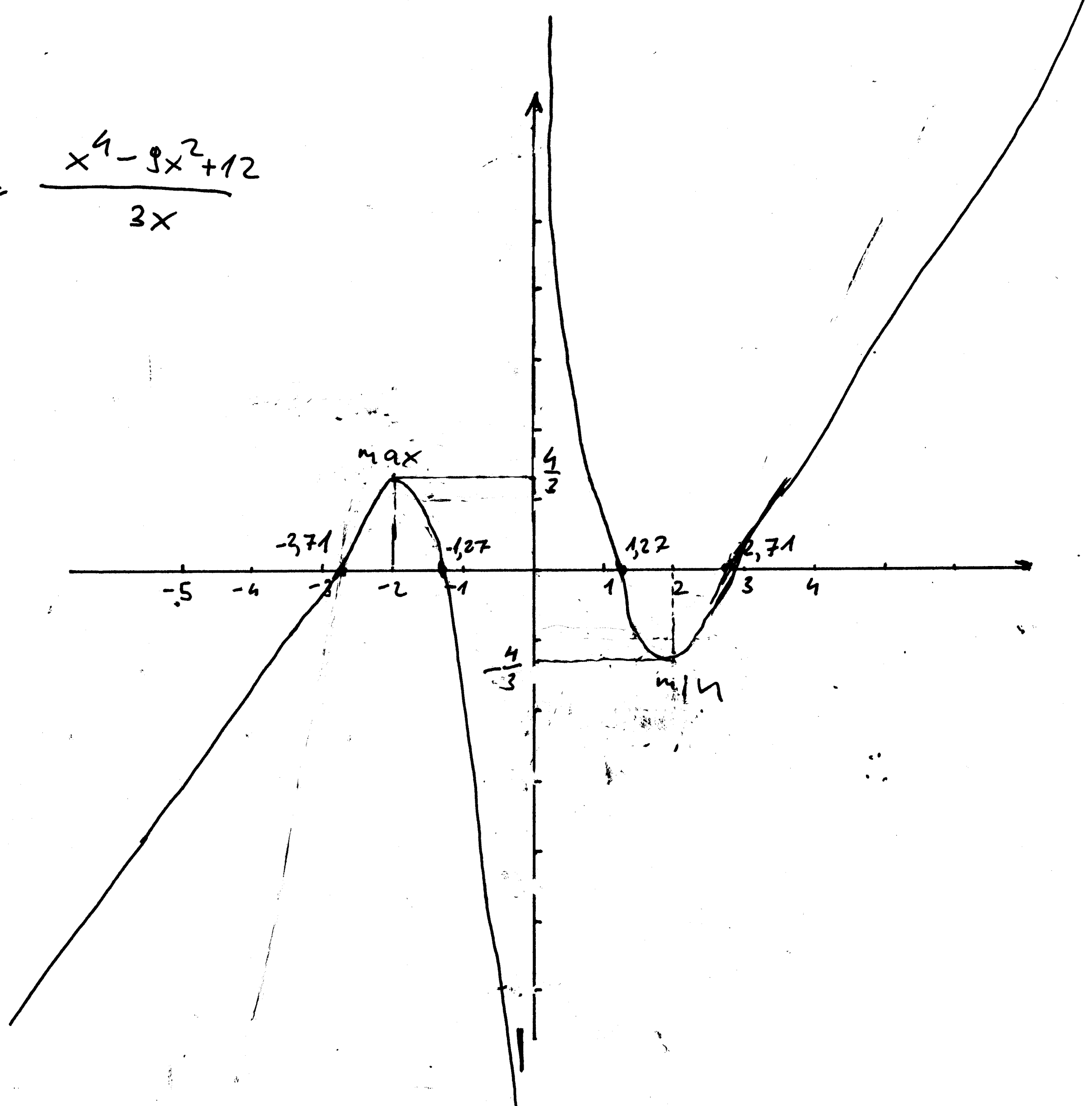
prevojne tačke i intervali konveksnosti i konkavnosti

$y'' = \left(x^2 - 3 - \frac{4}{x^2}\right)' = 2x - 4(-2)x^{-3} = 2x + \frac{8}{x^3}$

$y'' = \frac{2x^4 + 8}{x^3}$ kako je $2x^4 + 8 > 0 \ \forall x \Rightarrow$ f-ja nema prevojnih tački

za $x < 0$ $\cap$, a za $x > 0$ $\cup$

f-ja $y = \frac{x^4 - 9x^2 + 12}{3x}$



(#) Ispitati f-ju $y = \frac{ax+b}{x^2+x+1}$ i nacrtati joj grafik ako se zna da ona ima ekstrem u tački $T(1, \frac{2}{3})$.

Rj. $f(x) = \frac{ax+b}{x^2+x+1}$

$f(1) = \frac{2}{3} \Rightarrow \frac{a+b}{3} = \frac{2}{3}$

$a+b = 2$

$y' = \frac{a(x^2+x+1) - (ax+b)(2x+1)}{(x^2+x+1)^2}$

$y' = \frac{a(x^2+x+1) - (2ax^2+ax+2bx+b)}{(x^2+x+1)^2}$

$y' = \frac{-ax^2-2bx+a-b}{(x^2+x+1)^2}$

U stacionarnoj tački f-ja može imati ekstrem

$y' = 0 \Rightarrow -ax^2-2bx+a-b = 0$

$x = 1$

$-a-2b+a-b = 0$

$-3b = 0$

$b = 0, \quad a = 2$

$y = \frac{2x}{x^2+x+1}$

$y' = \frac{-2x^2+2}{(x^2+x+1)^2}$

$y' = (-2)\frac{x^2-1}{(x^2+x+1)^2}$

definiciono područje

$x^2+x+1 \neq 0$

f-ja je definisana za $\forall x$

nule, presjek sa x-osom, znak f-je

$y = 0 \Rightarrow 2x = 0 \Rightarrow x = 0$

$(0,0)$ je presjek sa y-osom i nula f-je

parnost (neparnost), periodičnost

$f(-x) = \frac{-2x}{x^2-x+1}$

f-ja nije ni parna ni neparna

f-ja nije periodična

kako je $x^2+x+1 > 0 \ \forall x$ to je

$y > 0$ za $x > 0$

$y < 0$ za $x < 0$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

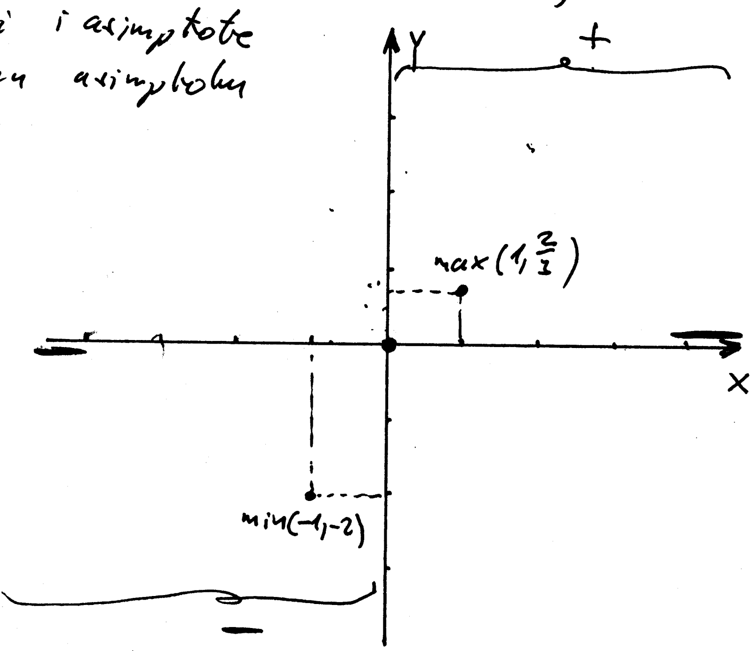
f-ja nema prekid $\Rightarrow$ f-ja nema vertikalnu asimptotu

$$\left.\begin{aligned}\lim_{x\to\infty} f(x) &= \lim_{x\to\infty} \frac{2x \ /:x}{x^2+x+1 \ /:x} = 0 \\ \lim_{x\to-\infty} f(x) &= \lim_{x\to-\infty} \frac{2x}{x^2+x+1} = 0\end{aligned}\right\} \Rightarrow$$

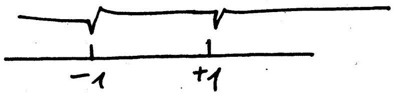
$\Rightarrow x = 0$ je H.A.

F-ja nema kosu asimptotu

Poslije ovog koraka počinjemo skicirati grafik f-je.



rast i opadanje



$$Y' = (-2)\,\frac{x^2-1}{(x^2+x+1)^2}$$

x	$(-\infty,-1)$	$(-1,1)$	$(1,+\infty)$
Y'	−	+	−
Y	↘	↗	↘

min max

$Y' = 0 \Rightarrow x = \pm 1$

ekstremi f-je

$$f(-1) = \frac{-2}{1-1+1} = -2$$

F-ja ima minimum u tački $P(-1,-2)$ i maksimum u tački $(1, \frac{2}{3})$

$$f(1) = \frac{2}{1+1+1} = \frac{2}{3}$$

prevojne tačke i intervali konveksnosti i konkavnosti

$$Y'' = (-2)\left(\frac{x^2-1}{(x^2+x+1)^2}\right)' = (-2)\,\frac{2x(x^2+x+1)^{\cancel{2}\,1} - (x^2-1)\,2\cancel{(x^2+x+1)}(2x+1)}{(x^2+x+1)^{\cancel{4}\,3}}$$

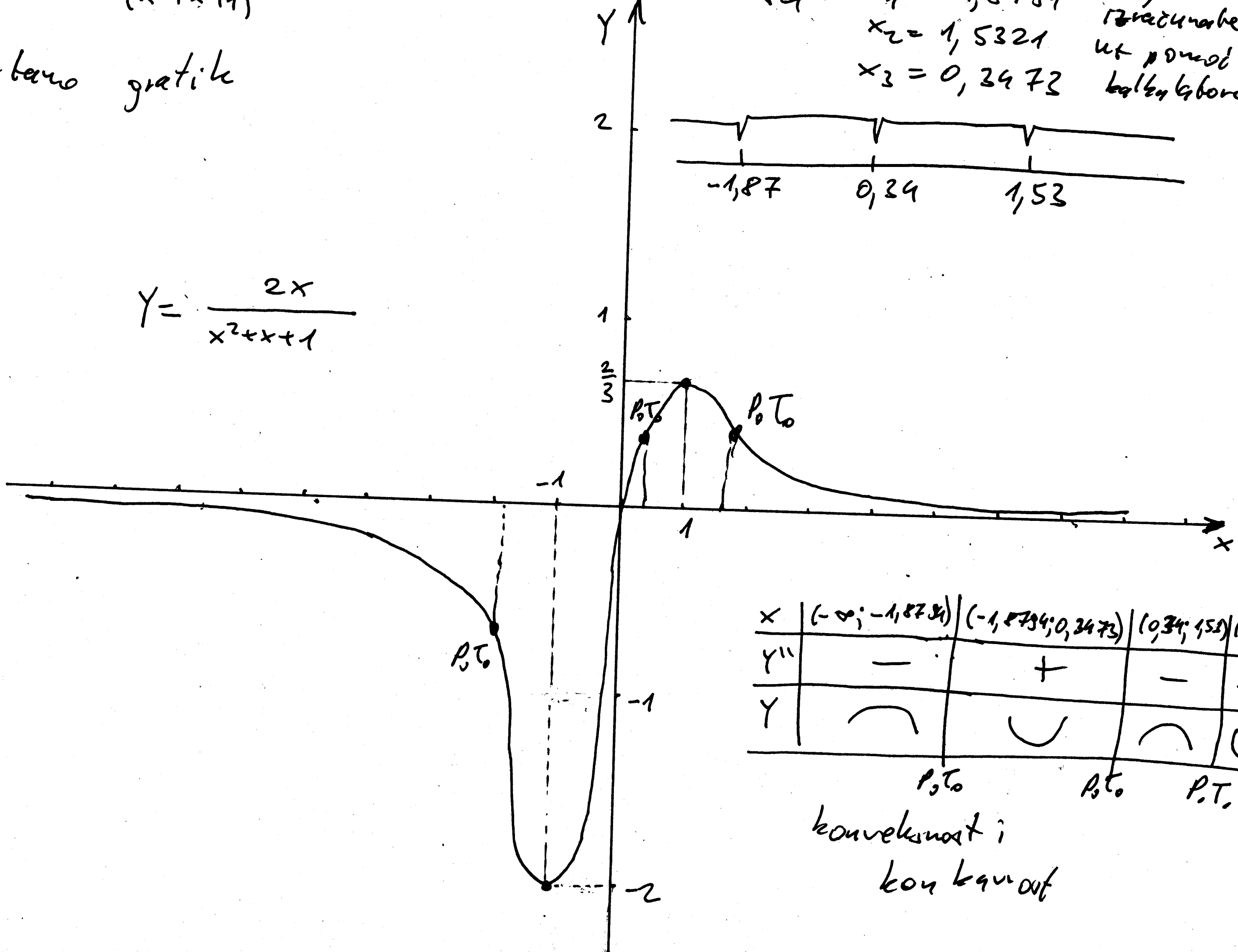
$$Y'' = (-2)\,\frac{2x^3+\cancel{2x^2}+2x-\cancel{4x^3}+2x-\cancel{2x^2}+2}{(x^2+x+1)^3} = (-2)\,\frac{-2x^3+x+2}{(x^2+x+1)^3} = (-2)\,\frac{(-2)(x^3-3x-1)}{(x^2+x+1)^3}$$

$$Y'' = 4\,\frac{x^3-3x-1}{(x^2+x+1)^3}$$

korjeni od x^3-3x-1 su $x_1 = -1,8794$, $x_2 = 1,5321$, $x_3 = 0,3473$ (vrijednosti izračunate uz pomoć kalkulatora)

crtamo grafik

$$Y = \frac{2x}{x^2+x+1}$$



x	$(-\infty;-1,8794)$	$(-1,8794;0,3473)$	$(0,34;1,53)$	$(1,53;\infty)$
Y''	−	+	−	+
Y	∩	∪	∩	∪

P_oT_o P_oT_o $P.T.$

konveksnost i konkavnost

(#) Ispitati f-ju i nacrtati joj grafik $y = x e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)}$

Rj. definiciono područje

$x \neq 0$

$\mathcal{D}: x \in \mathbb{R} \setminus \{0\}$

parnost (neparnost), periodičnost

$f(-x) = -x e^{\frac{1}{2}\left(1-\frac{1}{(-x)^2}\right)} = -x e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} = -f(-x)$

f-ja je neparna

f-ja nije periodična

nule, presjek sa y-osom, znak f-je

$f(0)$ nije definisano

f-ja ne sječe y-osu

$y \neq 0,\ \forall x \in \mathcal{D}$

$\left(e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} > 0\ \forall x\right)$

f-ja nema nulu

x	$(-\infty, 0)$	$(0, +\infty)$
y	$-$	$+$

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

za $x=0$ f-ja ima prekid

$\lim\limits_{x\to 0-} f(x) = \lim\limits_{x\to 0-} x e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} = (0-)\cdot e^{\frac{1}{2}(1-\infty)} = (0-) e^{-\infty} = \frac{0-}{e^{\infty}} = 0$

$\lim\limits_{x\to 0+} f(x) = \lim\limits_{x\to 0+} x e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} = (0+) e^{-\infty} = 0$

f-ja nema vertikalnu asimptotu

$\lim\limits_{x\to -\infty} f(x) = \lim\limits_{x\to -\infty} x e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} = (-\infty)\cdot e^{\frac{1}{2}} = -\infty$

$\lim\limits_{x\to \infty} f(x) = \lim\limits_{x\to \infty} x e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} = \infty \cdot e^{\frac{1}{2}} = \infty$

f-ja nema horizontalnu asimptotu

tražimo kosu asimptotu u obliku

$y = kx + n$

$k = \lim\limits_{x\to\infty} \frac{f(x)}{x} = \lim\limits_{x\to\infty} e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} = e^{\frac{1}{2}} = \sqrt{e}$

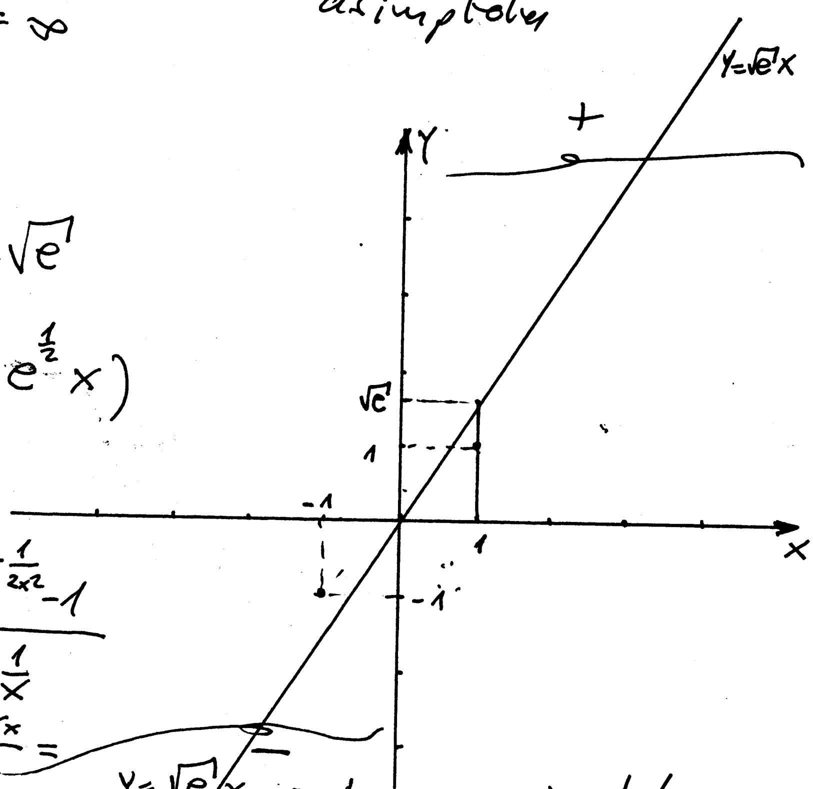
$n = \lim\limits_{x\to\infty} [f(x) - k\cdot x] = \lim\limits_{x\to\infty} \left(x e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} - e^{\frac{1}{2}} x\right)$

$= \lim\limits_{x\to\infty} x\left(e^{\frac{1}{2}\left(1-\frac{1}{x^2}\right)} - e^{\frac{1}{2}}\right) =$

$= \lim\limits_{x\to\infty} e^{\frac{1}{2}} x\left(e^{\frac{-1}{2x^2}} - 1\right) = \sqrt{e} \lim\limits_{x\to\infty} \frac{e^{-\frac{1}{2x^2}} - 1}{\frac{1}{x}}$

$\left(=\frac{0}{0}\right) \overset{L.P.}{=} \sqrt{e} \lim\limits_{x\to\infty} \frac{e^{-\frac{1}{2x^2}}\cdot\left(-\frac{1}{2}\right)(-2)\frac{1}{x^3}}{\frac{-1}{x^2}} =$

$= \sqrt{e} \lim\limits_{x\to\infty} \frac{-e^{-\frac{1}{2x^2}}}{x} = \sqrt{e}\cdot\frac{-1}{\infty} = 0$



$y = \sqrt{e}x$ je kosa asimptota

počinjemo sa skiciranjem grafika $\sqrt{e} \approx 1{,}64$

rast i opadanje

$(-\frac{1}{x^2})' = (-x^{-2})' = 2x^{-3} = \frac{2}{x^3}$

$$y' = \left(x e^{\frac{1}{2}(1-\frac{1}{x^2})}\right)' = e^{\frac{1}{2}(1-\frac{1}{x^2})} + x e^{\frac{1}{2}(1-\frac{1}{x^2})} \cdot \left(\frac{1}{2}(1-\frac{1}{x^2})\right)' =$$

$$= e^{\frac{1}{2}(1-\frac{1}{x^2})} + x e^{\frac{1}{2}(1-\frac{1}{x^2})} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{x^3} = e^{\frac{1}{2}(1-\frac{1}{x^2})}\left(1 + \frac{1}{x^2}\right)$$

$y' = 0$ akko $1 + \frac{1}{x^2} = 0$

$\frac{x^2+1}{x^2} = 0$

$y' > 0 \ \forall x \Rightarrow$ f-ja uvijek raste

f-ja nema ekstrema

prevojne tačke i intervali konveksnosti i konkavnosti

$$y'' = \left[e^{\frac{1}{2}(1-\frac{1}{x^2})}\left(1+\frac{1}{x^2}\right)\right]' = e^{\frac{1}{2}(1-\frac{1}{x^2})} \cdot \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{x^3}\left(1+\frac{1}{x^2}\right) + e^{\frac{1}{2}(1-\frac{1}{x^2})} \cdot \frac{-2}{x^3} =$$

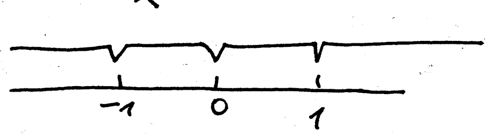
$$= e^{\frac{1}{2}(1-\frac{1}{x^2})}\left(\frac{1}{x^3} + \frac{1}{x^5} - \frac{2}{x^3}\right) = \left(\frac{1}{x^5} - \frac{1}{x^3}\right) e^{\frac{1}{2}(1-\frac{1}{x^2})}$$

$f(1) = 1 e^{\frac{1}{2}0} = 1$

$y'' = 0$ akko $\frac{1-x^2}{x^5} = 0 \Rightarrow 1 - x^2 = 0$

$x = \pm 1$

prekidi od y + nule od y''



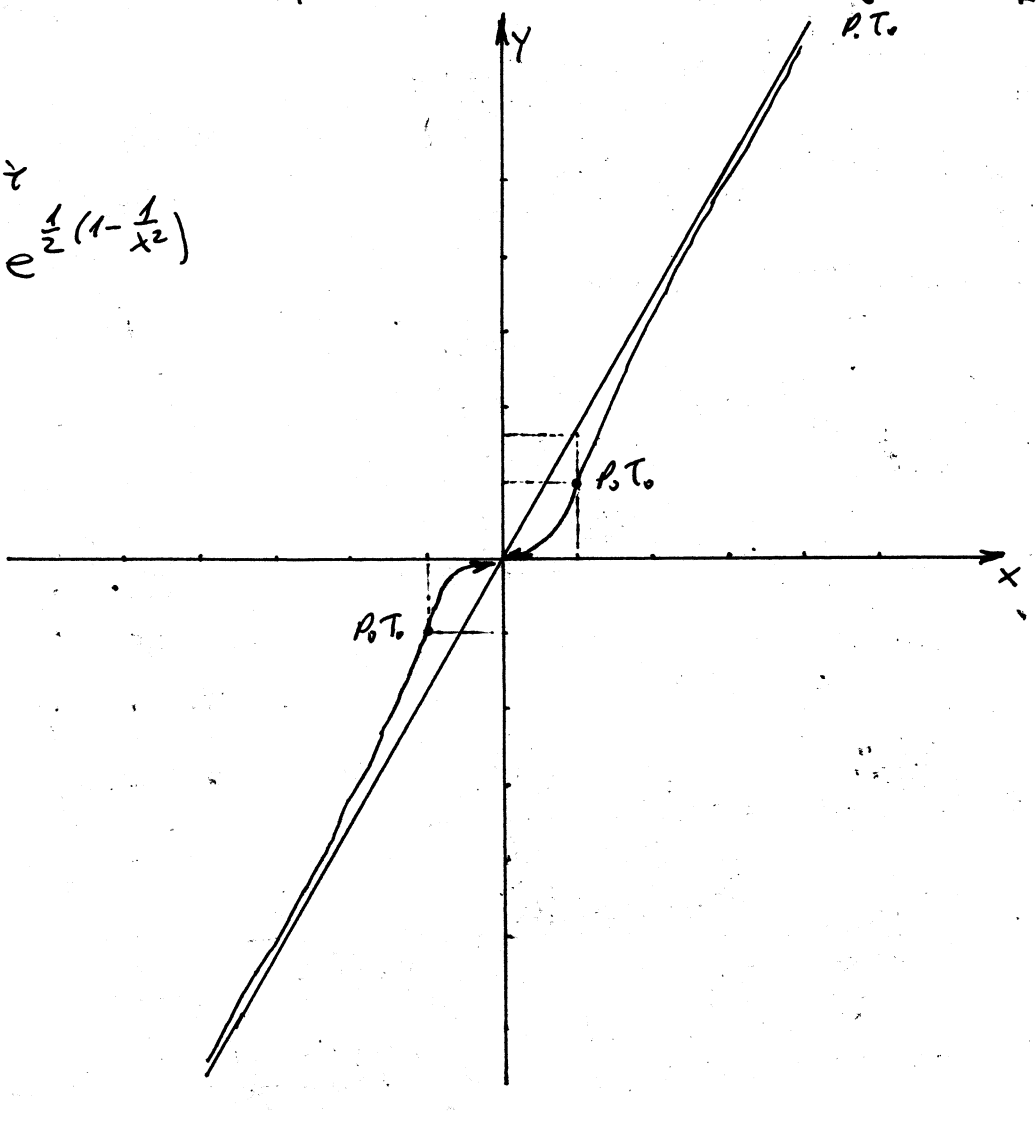
	(0, 1)	(1, +∞)
y''	+	−
y	∪	∩

P.T.

(1, 1) i (−1, −1) su prevojne tačke

graf. f-je

$y = x e^{\frac{1}{2}(1-\frac{1}{x^2})}$



(#) Ispitati f-ju i nacrtati joj grafik $y = \ln \dfrac{x^2-3x+2}{x^2+1}$.

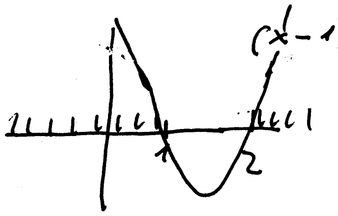
Rj: definiciono područje

Kako je $x^2+1>0 \quad \forall x \in \mathbb{R}$

to iz $\dfrac{x^2-3x+2}{x^2+1} > 0 \Rightarrow$

treba da bude $x^2-3x+2>0$

$(x-1)(x-2)>0$



$\mathcal{D}: x \in (-\infty, 1) \cup (2, +\infty)$

nule, presjek sa y-osom, znak

$y=0 \Rightarrow \ln \dfrac{x^2-3x+2}{x^2+1} = 0$

$\Rightarrow \dfrac{x^2-3x+2}{x^2+1} = 1 \quad / \cdot x^2+1$

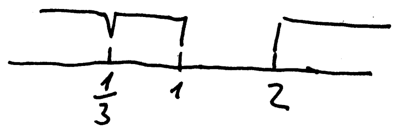
$\cancel{x^2}-3x+2 = \cancel{x^2}+1$

$3x = 1 \Rightarrow x = \frac{1}{3}$

$(\frac{1}{3}, 0)$ je nula f-je

$y(0) = \ln 2 \approx 0{,}6931$

$(0, \ln 2)$ je presjek sa y-osom



← prekidi y + nule y

x	$(-\infty, \frac{1}{3})$	$(\frac{1}{3}, 1)$	$(2, +\infty)$
y	+	−	−

znak f-je

parnost (neparnost), periodičnost

$\mathcal{D}$ nije simetrično $\Rightarrow$ f-ja nije ni parna ni neparna

f-ja nije periodična

ponašanje na krajevima intervala definisanosti i asimptote

f-ja ima prekid za $x=1$ i $x=2$

$$\lim_{x\to 1-0} f(x) = \lim_{x\to 1-0} \ln \frac{x^2-3x+2}{x^2+1} = \ln \frac{(1-0)^2-3(1-0)+2}{(1-0)^2+1} = \ln(0_+) = -\infty \Rightarrow$$

$\Rightarrow x=1$ je V. A. (sa lijeve str.)

$$\lim_{x\to 2+0} f(x) = \lim_{x\to 2+0} \ln \frac{x^2-3x+2}{x^2+1} = \ln(0+) = -\infty$$

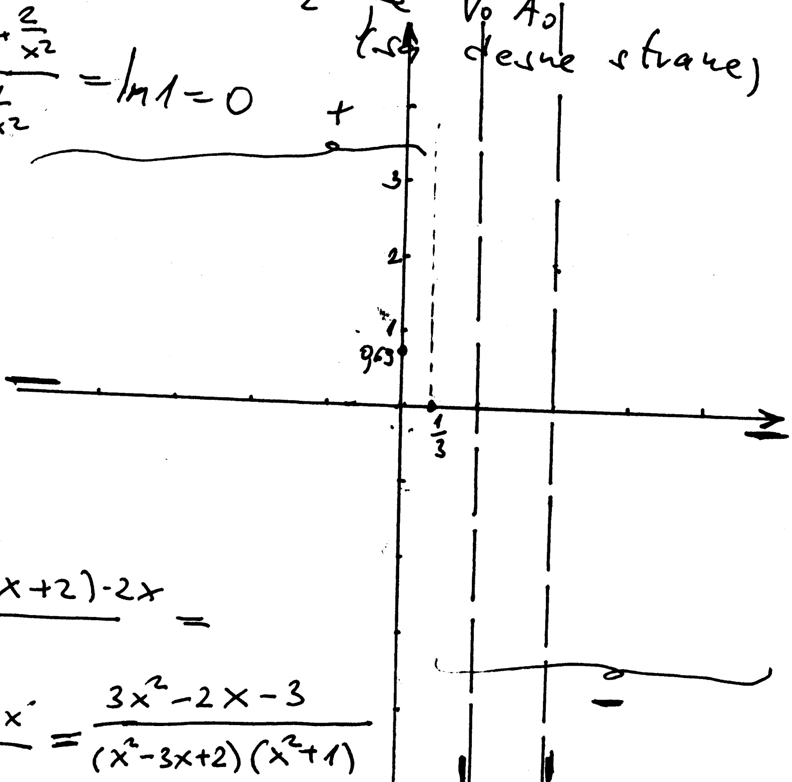
$\Rightarrow x=2$ je V. A. (sa desne strane)

$$\lim_{x\to\mp\infty} f(x) = \lim_{x\to\mp\infty} \ln \frac{x^2-3x+2}{x^2+1} = \lim_{x\to\mp\infty} \frac{1-\frac{3}{x}+\frac{2}{x^2}}{1+\frac{1}{x^2}} = \ln 1 = 0$$

$\Rightarrow y=0$ je H. A.

K. A. nema.

počinjemo sa skiciranjem grafa



rast i opadanje

$$y' = \frac{x^2+1}{x^2-3x+2} \cdot \left(\frac{x^2-3x+2}{x^2+1}\right)'$$

$$y' = \frac{\cancel{x^2+1}}{x^2-3x+2} \cdot \frac{(2x-3)(x^2+1)-(x^2-3x+2)\cdot 2x}{(x^2+1)^{\cancel{2}}} =$$

$$= \frac{\cancel{2x^3}+2x-3x^2-3-\cancel{2x^3}+6x^2-4x}{(x^2-3x+2)(x^2+1)} = \frac{3x^2-2x-3}{(x^2-3x+2)(x^2+1)}$$

$y' = 0 \Rightarrow 3x^2 - 2x - 3 = 0 \Rightarrow x_{1,2} = \dfrac{2 \pm \sqrt{4+36}}{6}$

$x_{1,2} = \dfrac{2 \pm 2\sqrt{10}}{6} = \dfrac{1 \pm \sqrt{10}}{3}$

$x_1 = \dfrac{1+\sqrt{10}}{3} \approx 1{,}387 \notin \mathcal{D}$

$x_2 = \dfrac{1-\sqrt{10}}{3} \approx -0{,}721 \in \mathcal{D}$

$-0{,}72 \quad 1 \quad x_2 \quad 2$

x	$\left(-\infty, \frac{1-\sqrt{10}}{3}\right)$	$\left(\frac{1-\sqrt{10}}{3}, 1\right)$	$(2, +\infty)$
y'	+	−	+
y	↗	↘	↗

max

ekstremi f-je

$f\left(\dfrac{1-\sqrt{10}}{3}\right) \approx 1{,}016$

F-ja ima maksimum u tački $(-0{,}72; 1{,}02)$

prevojne tačke i intervali konveksnosti i konkavnosti

$y'' = \left(\dfrac{3x^2-2x-3}{(x^2-3x+2)(x^2+1)}\right)' = \begin{matrix}\text{ZA} \\ \text{VJEŽBU} \\ \dots\end{matrix} = \dfrac{-6x^5+15x^4-30x^2+30x-13}{(x^2-3x+2)^2(x^2+1)^2}$

$y'' = 0$ akko $x = -1{,}5166$ (izračunato uz pomoć kalkulatora)

Kako je brojnik u y'' previše složen nije potrebno praviti tabelu konveksnosti i konkavnosti

grafik f-je

$y = \ln \dfrac{x^2-3x+2}{x^2+1}$